13th UTK Undergraduate Math Conference 2019

Abstracts (alphabetically by first author)

- **Austin Barringer** , Rhodes College (Talk #8)

Computation and Analysis of the Hilbert Series of Covariants for the Circle Action

The Hilbert series is a power series whose coefficients count the dimensions of the graded components of a graded ring, e.g. the polynomials of a given degree. In our case, the ring is the set of polynomials covariant under the circle action. The aim of this project is to compute an expression for the Hilbert series as well as the first two Laurent coefficients. This computation gives us precise information about our group, by telling us how many covariants we have of a certain degree. We will present a formula for the Hilbert series when the weight vector is not degenerate. We will also present formulas for the first and second Laurent series coefficients, and discuss progress toward implementing an algorithm on Mathematica to compute examples and test our formulas empirically. *Adviser: Christopher Seaton*

- **Amy Carpenter** , Lee University (#5)

Riding the Waves: How changes in a battery's electrical waveform can give clues to its health

Lithium batteries' capability of combusting under particular circumstances proves to be quite inconvenient for consumers. Lithium can build up on the anode forming dendrites. If the dendrite connects to the cathode, then the battery will short circuit. This ability to combust is a cause for concern, thus understanding the behavior would behoove us. However, as of now, there is no method of detection that does not destroy the battery in the process. Our collaborator at Oak Ridge National Laboratory is developing one possible technique, in which a small voltage is applied to a cell so that the resulting current can be measured. He has performed several of these electrochemical impedance spectroscopy (EIS) experiments on a circuit modeling a simple battery, varying the amplitude and waveform of the voltage for each experiment. When a sinusoidal voltage is applied, the graph of the flow of electrons versus time (the current) will also have a wave-like shape. However, complicated elements such as capacitors and diodes cause the graph of the current to consistent of the sum of several sine waves. We investigate the presence of higher harmonics and intermodulation (linear combinations of the frequencies of the input voltage) in the graph of the current. We show how the unique waveform of the current reflects the elements included in the circuit. In a real battery, analyzing the graph of the current could help determine which reactions are happening and whether the battery is functioning properly.
Advisers: Debra Gladden, Jason Schmurr

- **Rory Casey** , UNC Charlotte (#6)

Methods for Modelling Dark Matter Density on Local Galactic Scales

The nature of Dark Matter is perhaps one of the most interesting in the field of modern astrophysics. Among the theories offered for the nature of the existence of Dark Matter are WIMPs, MACHOs, and neutrinos. Each of these theories has unique features which will impact the models needed to analyze them. Before any differences in modeling can be determined, it is necessary to build a general model equation and a program capable of handling the necessary calculations. The purpose of this research was to build the necessary program and develop error constraints on the models. This research began by developing an algebraic N-body simulator capable of performing Dark Matter calculations and using the Virial Theorem to determine error bounds. In addition, the equation for the N-body simulation was converted into a differential equation simulator capable of using finite approximation methods to determine Dark Matter distributions more accurately (especially when those distributions cannot be approximated as point sources as is the case with neutrinos and WIMPs). N-body benchmarking problems are also tested which contributed to the observation of interesting patterns of behavior for dark matter. *Adviser: Xingjie Li*

- **Casey Dowdle** , Clemson (#10)

Using Dilated Convolution Neural Networks as a Generative Model

We wanted to know how one can use neural networks to model audio. We investigate the mechanisms behind Google DeepMind's WaveNet algorithm, which uses a dilated convolution neural network to generate audio that replicates the features of its training dataset. We look at how the WaveNet algorithm compares with other generative models. We find that the WaveNet algorithm can be less computationally expensive compared to other algorithms such as recurrent neural networks, but can still realistically replicate audio. We also show how the algorithm can be extended to generate simulations from time-dependent data. *Adviser: Eleanor Jenkins*

- **Ayesha Ejaz** and **Kristen Scheckelhoff** , UNC Greensboro (#4)
  Balancing the cost of infection: The effect of clean needle use on the spread of hepatitis C among injecting drug users

Hepatitis C is an infectious liver disease which contributes to an estimated 400,000 deaths each year. The disease is caused by the hepatitis C virus (HCV) and is spread by direct blood contact between infected and susceptible individuals. Despite the magnitude of its impact on human populations, hepatitis C receives relatively little scientific attention. In particular, studies targeting disease eradication in the populations most at risk -- injecting drug users -- are scarce. Here we construct a game-theoretic model to investigate the effect of clean needle use on the spread of HCV. Individual drug users, seeking to maximize their own payoff by weighing the cost of clean equipment against the cost of infection, may opt for a level of protection between 0 and 100%. We find that the spread of HCV in this population can theoretically be eliminated if individuals use sterile equipment approximately two-thirds of the time. Achieving this level of compliance, however, requires that the real and perceived costs of obtaining sterile equipment are essentially zero. *Adviser: Igor Erovenko*

- **Alan Gan** , UTK (#9)
  Modeling hunting and harvesting interactions between plants and their seed dispersers

The unsustainable harvest of wild animals and plants presents grave biodiversity conservation and livelihood challenges. In particular, non-timber forest products are crucial for the viability of the ecosystem in addition to the economic livelihood for millions of people who rely on them for income. Previous research on plant-animal harvesting has mainly focused on seed dispersal failure under defaunation, but neglects the hunting and harvesting interactions between plants and their seed dispersers. To that end, we investigate and represent this interaction by formulating a discrete-time mathematical model. Through various analysis of sensitivity and growth rate, we determine that adult survivorship under harvest as well as hunting are critical parameters for ensuring the population persistence of both the plant and their seed dispersers. Furthermore, we illustrate the broad application of our model by using the Brazil nut-agouti as an example system. Overall, we conclude that conjoined management of co-harvested species is crucial when determining conservation decisions. *Advisers: Charlotte Chang, Xingli Giam*

- **Cain Gantt** , Georgia College (#3)
  The Fourier Transform and Signal Processing

In this project, we explore the Fourier transform and its applications to signal processing. We begin from the definitions of the space of functions under consideration and several of its orthonormal bases, then summarize the Fourier transform and its properties. After that, we discuss the Convolution Theorem and its relationship to the physics behind problems in signal processing. Finally, we investigate the multidimensional Fourier transform; in particular, we consider the 2-dimensional transform and its use in image processing and other problems. We include an example of a typical image processing task and demonstrate how the Convolution Theorem is applied to obtain a solution. *Adviser: Hong Yue*

- **Emily Horner, Abby Baucom, Chase Toomey** , Lee University (Poster)
  Prove It: Arguments that Prospective Teachers Find Convincing

With the different ways students think about proof and argumentation in mathematics, research has shown that many students tend to lean towards one type of proof scheme and when given different types of arguments, students often choose empirical arguments to be the most convincing. Knowing this, it is important to determine the type of proof scheme that prospective teachers possess in order to evaluate what they deem as a convincing form of proof. Therefore, building upon the foundational research of Harel and Sowder, Stylianides, and others, this study continues to seek the evaluation of the proof schemes of prospective middle grades mathematics teachers. By conducting one-on-one interviews with eight prospective middle grades mathematics teachers enrolled in a proof-based geometry class, the purpose of this study is to evaluate which types of arguments the participants found to be most convincing. The

presentation of the findings will include not only a discussion about which arguments were most convincing but also discuss the implications of these findings for mathematics teacher preparation.
Adviser: Laura Singletary

- **Colleen Hulsey** , Rhodes College (#15)
Using Matrix Models to Predict Long-Term Population Dynamics of Two Tree Species
Native tree species serve important ecological and economic roles, but they are threatened by environmental disturbances including competition with invasive species. Sugar maple (Acer saccharum) is a native tree in Tennessee, important for its lumber, sap, and pollution control. Tree-of-heaven (Ailanthus altissima) is an invasive species in Tennessee, characterized by its rapid growth which chokes out native species. I used matrix models to examine their population growth rate, stable population structures, and sensitivity at different age classes. I also examined the range of these outputs given uncertainty in the parameters. *Adviser: Erin Bodine*

- **Abbey Johnson** , UTK (#7)
Advanced MCMC parameter estimation of Sphagnum gross primary production in the S1bog at Marcell Experimental Forest
Peatlands harbor vast stockpiles of carbon (approximately one-fifth to one-third of global soil carbon), which are susceptible to recent and future climate change. Particularly in northern wetlands, increasing temperature and vapor pressure deficit could induce a large feedback of CO_2 and CH_4, as these terrestrial carbon stockpiles degrade and return to the atmosphere. Sphagnum gross primary production (GPP) is a major entry point of carbon into peatland ecosystems, making it a central component of peatland carbon cycling. This study evaluates alternative mechanistic hypotheses, represented in a process-based model, for the drivers of seasonality in Sphagnum GPP. To rigorously evaluate the alternative hypotheses, parameters were estimated against Sphagnum GPP data using Markov chain Monte Carlo (MCMC) algorithms developed in a flexible modelling software, the Multi-Assumption Architecture and Testbed (MAAT). Predictions from the optimized models (hypotheses) were then evaluated against a validation dataset. Data were collected at the Spruce and Peatland Responses Under Changing Environments (SPRUCE) experiment sited in the Marcell Experimental Forest in northern Minnesota. Sphagnum magellanicum GPP fluxes were calculated from hourly measurements of gas exchange in LI¬8100s situated in hollows throughout the growing seasons from 2014-2018. This study applied the developed MCMC algorithms to compare model fit between two alternate Sphagnum GPP models that represent two hypotheses – constant Shoot Area Index (SAI) and dynamic SAI. In this analysis, SAI is the photosynthesizing tissue area per unit ground area. Constant SAI assumes constant SAI during the growing season, while dynamic SAI assumes an interaction between photosynthesizing tissue surface area (i.e., SAI) and fluctuating water table height, reflecting the idea that submerged tissue is not photosynthetically active. The MCMC parameter estimation processes implemented in MAAT formally showed that the dynamic SAI model better explained the seasonal dynamics in the estimated GPP. Thus, this study demonstrated that accurate models of Sphagnum GPP at the Marcell Experimental Forest should incorporate the interaction between changing water table levels and the Sphagnum surface. In a broader context, the parameter estimation methods developed by this study enable the discovery of the most parsimonious model of Sphagnum GPP from a candidate set of models. By determining which Sphagnum GPP model is better equipped at describing a set of observed data, the most parsimonious model can subsequently be chosen for use in a predictive setting, thereby contributing to carbon cycle and climate change research. *Adviser: Anthony Walker*

- **Benjamin Jones** , U Alabama (#14)
Numerical algorithms for electrostatic analysis of solvated biomolecules
The solvation energy of a molecule is the energy absorbed or released when that molecule is dissolved in a solvent. Computational biologists use the solvation energy of macromolecules, such as proteins, to determine their stability for protein folding and their hydrophobicity for drug delivery. The solvation energy of a protein can be found by solving the nonlinear Poisson-Boltzmann equation (PBE). The PBE is computationally intensive to solve for large proteins. Therefore, efficient, accurate, and stable algorithms are desired. The computation requires knowledge of a molecular surface. In this work, the Solvent Excluded Surface (SES) is used in the Poisson-Boltzmann model. The generation of this surface may lead to instabilities in the computation, particularly when using values that have a jump condition at the protein

interface. A standard SES generation tool is MSMS, which is known to cause some numerical instabilities. The Eulerian solvent excluded surface (ESES) tool was recently created and may address some of these instabilities. In this work, the impact of the different MSMS and ESES surfaces on the stability of an Alternating Direction Implicit (ADI) method are compared. *Adviser: Shan Zhao*

- **Farzana Nasrin** , UTK
  Opening Lecture: ***Topological Machine Learning in Material Science***

Recent advances in material science have led to the era of enormous databases of materials for different applications. Analyzing and classifying such large and complex datasets are generally challenging. Topological data analysis, that builds on techniques from topology, is a natural fit for this application. Persistence diagram is a powerful tool originated in topological data analysis that allows retrieval of important topological and geometrical features latent in a dataset. Data analysis and classification involving persistence diagrams have been applied in numerous applications such as action recognition, handwriting analysis, shape study, image analysis, sensor network, and signal analysis. In this talk we will provide a brief introduction of topological data analysis, focusing primarily on persistence diagrams. Our goal is to provide a supervised machine learning algorithm, the classification, on the space of persistence diagrams. This framework is applicable to a wide variety of datasets. We will present application in material science, specially classification of crystal structures of High Entropy Alloys.

- **Connor O'Ryan, Blayne Carroll** , Lee University (#12)
  The Development of a Model for a Battery to Predict Dendrite Growth

Under the direction of our contact at Oak Ridge National Laboratory, we study dendrite buildup within lithium ion batteries. This dendrite is the formation of solid lithium metal on the electrodes of the battery that eventually causes battery failure. This failure has resulted in explosions and other safety issues, therefore we would like a way to predict this occurrence and establish safeguards. First we create a simulation of a real electrical circuit provided to us by Dr. Sacci. Our group has used computer software systems to recreate this circuit. We found an online circuit simulator system that allows us to build out the circuit to simulate a battery by replicating the circuits structure, then modifying the elements behavior. This electrical circuit was used to model what happens inside of the ideal lithium ion battery. Our simulation is an attempt to replicate the graphs produced by Dr. Saccis experiment. The model has a graphical output, which depicts the current through the system at any given time, which we then compare to the provided data. This allows us to look for discrepancies between experimental data and our models generated data. Secondly, we would like to be able to fit this current vs time curve to an equation. Our hope by fitting this curve to an equation is to predict when a given lithium ion battery will malfunction. Ideally this model could be implemented as an added safety feature in lithium ion batteries, designed to predict how near to a short circuit the batteries are before their users are in any danger. *Advisers: Debra Gladden, Jason Schmurr*

- **Moises Ponce, Nicolette Gordon** , Lee University (#2)
  The Role of Proof Schemes in Preparing Future Educators

Previous research has found that students have varying schemes for thinking about proof in mathematics. Little research, however, exists that specifically examines the proof schemes of teachers, though teachers may influence their students' mathematical proof schemes. This study investigates prospective middle grades mathematics teachers' proof schemes through the analysis of 9 interviews while they are currently enrolled in an undergraduate proof-based geometry course. These interviews focus on prospective teachers' motivation for choosing the field, their beliefs and thoughts about mathematics, as well as their own proof schemes and their evaluation of student work. The findings presented will detail patterns in the prospective teachers' proof schemes and provide implications for the preparation of future teachers in content bearing courses. *Adviser: Laura Singletary*

- **Rebekah Petrosky, John Iluno, Ethan Large** , Lee University (#11)
  Modeling Impedance Change in a Simulated Lithium Ion Battery

The purpose of this project is to increase the safety of lithium ion (Li-Ion) batteries. The process of repeatedly charging and discharging this type of battery can cause a significant buildup of lithium in the form of dendrites on the porous diodes. Enough buildup connects the electrodes and causes a catastrophic short circuit. We cannot directly measure the dendrite formation, but we are working to understand how it affects the batteries' impedance. Our contact at Oak Ridge National Laboratory has provided data from a battery-like circuit in several different environments. To produce this data, he applied a potential difference to the circuit and recorded the resulting impedance using Electrochemical Impedance Spectroscopy (EIS). Our focus is on statistical analysis of patterns in the data. We have observed how the impedance of the circuit changes with respect to the independent variables of time and potential. We intend to apply this observation to create a mathematical model of battery behavior. Our current analysis suggests that we can model the circuit's behavior as a combination of sine waves found by performing Fourier Transforms on the data. *Advisers: Debra Gladden, Jason Schmurr*

- **Kinlee Pruitt, Ky'Anna Arthurton, Madison Mabe** , Lee University (#16)
  Visualizing the Conceptions Within Proof

The learning of proof within the context of the middle grades' mathematics classroom can be influenced by the teacher's understanding of mathematics and the validation techniques utilized within the classroom. Given that proof is an influential aspect of mathematics, this deserves further study. There are two perspectives of this issue- the student and the teacher. Past research has identified how students conceptualize the validity of proofs. Our study extends this line of inquiry and examines teachers' conceptions of proof as well as their corresponding proof schemes. Data collection involved individual audio-recorded interviews of prospective middle grades mathematics teachers who are currently enrolled in a proof-based geometry class. In the interviews, the participants articulated their background within mathematics, their thoughts regarding proof and the discipline of mathematics, constructed mathematical proofs, and evaluated various pre-determined proofs. For this presentation, prospective teachers' conceptions of mathematics and proof and how these conceptions are visible in what they consider as valid argumentation will be explored. *Adviser: Laura Singletary*

- **Sara Wonner** and **Chapel Rice** , UTK (#1)
  Price of Gold via Machine Learning and Least Squares

Gold is important to the world economy, and it has become a common investment and store of value. Investors will often buy gold, due to its intrinsic value, during economic uncertainty. This causes the price of gold to fluctuate proportionally to economic conditions and trends. Predicting the price of gold can have financial benefits, and it can predict future economic conditions. There are several macroeconomic trends that correlate with the price of gold, such as money supply, defence spending, and unemployment rate. This data was utilized to create and refine a model to predict the price of gold. Methods utilized include least squares methods and machine learning. The most accurate least squares model resulted in an average error of $65.70. Different machine learning regression methods were employed to arrive at a feasible model. The current, most accurate machine learning model was created utilizing the Gaussian Process Regression method and yielded an average error of $22.72. *Adviser: V Alexiades*

- **Chuanlong Zhang** , MTSU (#13)
  Research about Loss Reserving Method in P&C Insurance

In insurance industry, actuaries determine the present value of future claim payments. By law, insurance companies are required to set a fund for their future liabilities, called loss reserving. By setting aside reserves, the insured is guaranteed that they could get payments due to accidents, even if the insurer were insolvent. Reserves should not be so high that would cause unreasonably high premiums, nor too low to provide payments for the insured. So, projecting future losses and setting appropriate loss reserves are two of most important tasks for actuaries. As for life insurance, there is a classical and widely-accepted method to calculate loss reserves in life contingency models. However, actuaries are still looking for a universal method for property and casualty insurance. Hence, recognizing the basic assumptions, principles, practicability, advantages and disadvantages among different loss reserving models is necessary in future research for practical and advanced methods. *Adviser: Don Hong*