Reduction of Order

Spse we have $y'' + p(t)y' + q(t)y = 0$ ✶

and we know (somehow) one solution, y_1 over some interval I. Then

$$* \quad y_2 = y_1 \int \frac{e^{-\int p(t)\,dt}}{y_1^2}\,dt \quad \text{is a}$$

second LI solution.

Pf: Try to mimic the discussion for var. of param.

We know $y = c_1 y_1$ is a solution, replace c_1 with $v(t) \equiv v$

$\therefore \quad y_2 = v y_1 \Rightarrow y_2' = v y_1' + v' y_1$

$y_2'' = v y_1'' + v' y_1' + v' y_1' + v'' y_1$

Then plug into $y'' + p(t) y' + q(t) y = 0$

This given

$$(vy_1''+2v'y_1'+v''y_1)+p(vy_1'+v'y_1)+qvy_1=0$$

$$v(\underbrace{y_1''+py_1'+qy_1}_{0})+y_1v''+(2y_1'+y_1)v'=0$$

Since y_1 is a solution to $(*)$ so we have

$$y_1v''+(2y_1'+py_1)v'=0$$

Let $w=v'$

$w'=v''$ Then

$$y_1w'+(2y_1'+py_1)w=0$$

and this is separable!