Ex 2 Solve $y'' + 4y' - 5y = te^t$, $y(0)=1$, $y'(0)=0$

Method

Apply Laplace Transform to both sides of the equation,

$$\mathcal{L}\{y'' + 4y' - 5y\} = \mathcal{L}\{te^t\}$$

Now $\mathcal{L}\{y''(t)\} = s^2 Y(s) - s\,y(0) - y'(0)$

$$= s^2 Y(s) - s - \phi = s^2 Y(s) - s$$

$$4\mathcal{L}\{y'(t)\} = 4\left[sY(s) - y(0)\right] = 4\left[sY(s) - 1\right]$$

$$= 4sY(s) - 4$$

$-5\mathcal{L}\{y(t)\} = -5Y(s)$, So the LT of the left side is

$$s^2 Y(s) - s + 4sY(s) - 4 - 5Y(s)$$

$$(s^2 + 4s - 5)Y(s) - s - 4$$

For the right side of the equation we need

$$\mathcal{L}\{te^t\} = \frac{1}{(s-1)^2} \qquad \underline{\#\ 17}$$

So now we have:

$$(s^2+4s-5)Y(s) - s - 4 = \frac{1}{(s-1)^2}$$ Then

$$(s^2+4s-5)Y(s) = \frac{1}{(s-1)^2} + s + 4$$

$$(s^2+4s-5)Y(s) = \frac{1 + (s+4)(s-1)^2}{(s-1)^2}$$ Then

$$Y(s) = \frac{1 + (s+4)(s-1)^2}{(s-1)^3(s+5)}$$

Now we want to apply Inv. LT to both sides

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

we need $\mathcal{L}^{-1}\left\{\frac{1 + (s+4)(s-1)^2}{(s-1)^3(s+5)}\right\}$

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We now need to find the PFD of

$$\frac{1+(s+4)(s-1)^2}{(s-1)^3(s+5)} = \frac{\overset{\frac{181}{216}}{A}}{s-1} + \frac{\overset{-\frac{1}{36}}{B}}{(s-1)^2} + \frac{\overset{\frac{1}{6}}{C}}{(s-1)^3} + \frac{\overset{\frac{35}{216}}{D}}{s+5}$$

Let $s=2 \Rightarrow 216A + 216B = 175$

Let $s=0 \Rightarrow -216A + 216B = -187$

Solving gives $B = -\frac{1}{36}$ & $A = \frac{181}{216}$

$$\therefore \quad y(t) = \frac{181}{216}e^{t} - \frac{1}{36}te^{t} + \frac{35}{216}e^{-5t} + \underset{*}{\frac{1}{6}\cdot\frac{1}{2}t^2e^{t}}$$

$$y(t) = \frac{181}{216}e^{t} - \frac{1}{36}te^{t} + \frac{35}{216}e^{-5t} + \frac{1}{12}t^2e^{t}$$

For * Note we need $\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^3}\right\}$

however by #17

$$\mathcal{L}\{e^{at}t^n\} = \frac{n!}{(s-a)^{n+1}}$$

in our denominator we have $(s-1)^3$

$\therefore n+1=3 \Rightarrow \underline{n=2}$

So in the numerator we need $2! = 2$

so mult. by $1 = \frac{2}{2} = \frac{1}{2}\cdot\underline{\underline{2}}$

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#4 Solve $y'' + 6y' + 5y = 12e^{t}$ $\quad y(0) = -1$, $y'(0) = 7$

take $\mathcal{L}T$ of both sides

$$\left(s^2 Y(s) - s\,y(0) - y'(0)\right) + 6\left(s\,Y(s) - y(0)\right)$$

$$+ 5Y(s) = \frac{12}{s-1}$$

$$\left(s^2 Y(s) + s - 7\right) + 6\left(s\,Y(s) + 1\right) + 5Y(s) = \frac{12}{s-1}$$

$$\left(s^2 + 6s + 5\right)Y(s) + s - 1 = \frac{12}{s-1}$$

$$\left(s^2 + 6s + 5\right)Y(s) = \frac{12}{s-1} + 1 - s = \frac{12}{s-1} - (s-1)$$

$$\left(s^2 + 6s + 5\right)Y(s) = \frac{12 - (s-1)^2}{s-1}$$

$$Y(s) = \frac{12 - (s-1)^2}{(s-1)(s^2 + 6s + 5)}$$

$$Y(s) = \frac{12 - (s-1)^2}{(s-1)(s+5)(s+1)}$$

Now find PFD of right side.

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$$\frac{12-(s-1)^2}{(s-1)(s+5)(s+1)} = \frac{\overset{1}{A}}{s-1} + \frac{\overset{-1}{B}}{s+5} + \frac{\overset{-1}{C}}{s+1}$$

So

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\underline{y(t) = e^{t} - e^{-5t} - e^{-t}}$$