7.4 Inverse LT

Def: Given $F(s)$, if there is an $f(t)$ continuous on $[0, \infty)$ satisfying

$$\mathcal{L}\{f(t)\} = F(s) \text{ then}$$

$f(t)$ is the inverse LT of $F(s)$

denoted $\mathcal{L}^{-1}\{F(s)\} = f(t)$.

The inverse LT is <u>Linear</u>.

i) $\mathcal{L}^{-1}\{F_1(s) + F_2(s)\} = \mathcal{L}^{-1}\{F_1(s)\} + \mathcal{L}^{-1}\{F_2(s)\}$

ii) $\mathcal{L}^{-1}\{\alpha F(s)\} = \alpha \mathcal{L}^{-1}\{F(s)\}$.

Ex 2 (pg 368) Very important to note procedure.

Find $\mathcal{L}^{-1}\left\{\frac{5}{s-6} - \frac{6s}{s^2+9} + \frac{3}{2s^2+8s+10}\right\} =$

$$5\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\} - 6\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{2s^2+8s+10}\right\}$$

$$5\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\} = 5e^{6t}$$ <u>#15 table</u>

$$-6\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = -6\cos(3t)$$ # 25 table

Now

$$2s^2+8s+10 = 2(s^2+4s+4)+10-8$$
$$= 2(s+2)^2+2 = 2\left((s+2)^2+1\right)$$

$$\therefore\ 3\mathcal{L}^{-1}\left\{\frac{1}{2s^2+8s+10}\right\} = \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}$$

$$= \frac{3}{2}e^{-2t}\sin(t)$$ # 26 table

So this gives

$$\mathcal{L}^{-1}\left\{\frac{5}{s-6} - \frac{6s}{s^2+9} + \frac{3}{2s^2+8s+10}\right\} = 5e^{6t} - 6\cos(3t) + \frac{3}{2}e^{-2t}\sin(t)$$

Ex 4 Find $\mathcal{L}^{-1}\left\{\frac{3s+2}{s^2+2s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{3s+2}{(s+1)^2+9}\right\}$

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we need to use formulas

26 : $\frac{b}{(s-a)^2+b^2} \longrightarrow e^{at}\sin(bt)$

27 : $\frac{s-a}{(s-a)^2+b^2} \longrightarrow e^{at}\cos(bt)$

we need to change given numerator $3s+2$ into multiples of b & $s-a$ in #26 & #27

So a partial fraction decomposition is needed, but in a different form than from Cal II

$$\frac{3s+2}{(s+1)^2+9} = \frac{A\,(s+1)}{(s+1)^2+9} + \frac{B\cdot 3}{(s+1)^2+9}$$

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$$\frac{3s+2}{(s+1)^2+9} = \frac{A(s+1)}{(s+1)^2+9} + \frac{B \cdot 3}{(s+1)^2+9}$$

let $s=0$ $\quad \frac{2}{10} = \frac{A}{10} + \frac{3B}{10} \Rightarrow \boxed{A+3B=2}$

let $s=-1$ $\quad -\frac{1}{9} = 0 + \frac{3B}{9} \Rightarrow -1 = 3B$

$$\boxed{-\tfrac{1}{3} = B}$$

Then $A - 1 = 2$

$$\boxed{A = 3}$$

So we have

$$\mathcal{L}^{-1}\left\{\frac{3s+2}{(s+1)^2+9}\right\} = 3\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+9}\right\}$$

$$= 3e^{-t}\cos(3t) - \frac{1}{3}e^{-t}\sin(3t)$$

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~~#22~~

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$$\mathcal{L}^{-1}\left\{\frac{s+11}{(s-1)(s+3)}\right\}$$

we need

$$\frac{s+11}{(s-1)(s+3)} = \frac{\overset{\frac{12}{4}=3}{A}}{s-1} + \frac{\overset{\frac{8}{-4}=-2}{B}}{s+3}$$

we want to use #15 table

we have

$$\mathcal{L}^{-1}\left\{\frac{s+11}{(s-1)(s+3)}\right\} = 3\,\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= 3e^{t} - 2e^{-3t}$$

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(modified)

$$\mathcal{L}^{-1}\left\{\frac{-2s^2-3s-2}{s(s+1)^2}\right\}$$

$$\frac{-2s^2-3s-2}{s(s+1)^2} = \frac{\overset{-2}{A}}{s} + \frac{\overset{\phi}{B}}{s+1} + \frac{\overset{1}{C}}{(s+1)^2}$$

Let $s=1 \Rightarrow$

$$\frac{-7}{4} = -2 + \frac{B}{2} + \frac{1}{4}$$

$$-7 = -8 + 2B + 1$$

$$1 = 2B + 1$$

$$0 = B$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{-2s^2-3s-2}{s(s+1)^2}\right\} = -2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

\# 17 table

$$= -2 + te^{-t}$$

\#28 Find $\mathcal{L}^{-1}\{F(s)\}$ given

$$s^2F(s) + sF(s) - 6F(s) = \frac{s^2+4}{s^2+s}$$

$$F(s)\left(s^2+s-6\right) = \frac{s^2+4}{s^2+s}$$

$$F(s) = \frac{s^2+4}{(s^2+s)(s^2+s-6)} = \frac{s^2+4}{s(s+1)\left((s+\frac{1}{2})^2-\frac{25}{4}\right)}$$

Now want

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{s^2+4}{s(s+1)\left((s+\frac{1}{2})^2-\frac{25}{4}\right)}\right\}$$

7.4 $\mathcal{L}^{-1}\{F(s)\} = f(t)$ Now we need

$$\mathcal{L}^{-1}\left\{ \frac{s^2+4}{s(s+1)\left((s+\frac{1}{2})^2 - \frac{25}{4}\right)} \right\}$$

$$\frac{s^2+4}{s(s+1)\left((s+\frac{1}{2})^2 - \frac{25}{4}\right)} = \frac{\overset{-2/3}{A}}{s} + \frac{\overset{5/6}{B}}{s+1} + \frac{C\left(\frac{5}{2}\right)}{(s+\frac{1}{2})^2 - \frac{25}{4}} + \frac{D\left(s+\frac{1}{2}\right)}{(s+\frac{1}{2})^2 - \frac{25}{4}}$$

let $s=1$ this gives (eventually) $\boxed{15C + 9D = 9}$

let $s = -\frac{1}{2}$ giving (eventually) $C = \frac{7}{10} \Rightarrow D = -\frac{1}{6}$

$$\therefore \mathcal{L}^{-1}\{F(s)\} = f(t) = -\frac{2}{3} + \frac{5}{6}e^{-t} + \frac{7}{10}e^{-\frac{1}{2}t}\sinh\left(\tfrac{5}{2}t\right) - \frac{1}{6}e^{-\frac{1}{2}t}\cosh\left(\tfrac{5}{2}t\right)$$

whew!

Now would probably rewrite the last two terms due to def. of hyperbolic trig. funs.