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#21 $y'' - 2y' + y = \cos t - \sin t$

$$\begin{cases} y(0) = 1 \\ y'(0) = 3 \end{cases}$$

$-2(sY(s) - 1)$

$$\left(s^2 Y(s) - s\overset{1}{y(0)} - \overset{3}{y'(0)}\right) - 2\left(sY(s) - \overset{1}{y(0)}\right) + Y(s) = \frac{s-1}{s^2+1}$$

$$(s^2 - 2s + 1)Y(s) - s - 1 = \frac{s-1}{s^2+1}$$

$$\underbrace{(s^2-2s+1)}_{(s-1)^2} Y(s) = \frac{s-1}{s^2+1} + s + 1 = \frac{s-1 + (s+1)(s^2+1)}{s^2+1}$$

$$Y(s) = \frac{s-1 + (s+1)(s^2+1)}{(s^2+1)(s-1)^2}$$

$$\frac{s-1 + (s+1)(s^2+1)}{(s^2+1)(s-1)^2} = \frac{\overset{3/2}{A}}{s-1} + \frac{\overset{2}{B}}{(s-1)^2} + \frac{\overset{-1/2}{C}\,s}{s^2+1} + \frac{\overset{-1/2}{D}}{s^2+1}$$

let $s = 0$ $\quad \frac{-1+1}{1 \cdot 1} \Rightarrow 0 = -A + 2 + D$ $\quad \boxed{-A + D = -2}$

let $s = -1 \Rightarrow \quad \frac{-2}{8} = \frac{A}{-2} + \frac{1}{2} - \frac{C}{2} + \frac{D}{2}$

$$-2 = -4A + 4 - 4C + 4D \quad \boxed{-4A - 4C + 4D = -6}$$

let $s = 2 \Rightarrow \dfrac{1 + 3(5)}{5} = A + 2 + \dfrac{2C}{5} + \dfrac{D}{5}$

$$\frac{16}{5} = A + 2 + \frac{2C}{5} + \frac{D}{5}$$

$$16 = 5A + 10 + 2C + D$$

$$(2)\left\{\begin{aligned} 5A + 2C + D &= 6 \\ -4A - 4C + 4D &= -6 \\ -A + D &= -2 \end{aligned}\right.$$

$$\begin{aligned} 10A + \cancel{4C} + 2D &= 12 \\ -4A - \cancel{4C} + 4D &= -6 \\ \hline 6A + 6D &= 6 \end{aligned}$$

$$A + D = 1$$

$$\begin{aligned} -A + D &= -2 \\ A + D &= 1 \\ \hline 2D &= -1 \end{aligned}$$

$$D = -\tfrac{1}{2}$$

$$A - \tfrac{1}{2} = 1 \Rightarrow A = \tfrac{3}{2}$$

$$-4\left(\tfrac{3}{2}\right) - 4C + 4\left(-\tfrac{1}{2}\right) = -6$$

$$\cancel{-6} - 4C - 2 = \cancel{-6}$$

$$-4C = 2$$

$$C = -\tfrac{1}{2}$$

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$$Y(s) = \frac{3}{2}\cdot\frac{1}{s-1} + \frac{2}{(s-1)^2} - \frac{1}{2}\,\frac{s}{s^2+1} - \frac{1}{2}\,\frac{1}{s^2+1}$$

$$\boxed{y(t) = \frac{3}{2}e^{t} + 2te^{t} - \frac{1}{2}\cos t - \frac{1}{2}\sin t}$$

#17 $\quad e^{at}t^{n} = \dfrac{n!}{(s-a)^{n+1}}, \quad n \in \mathbb{Z}^{+}$