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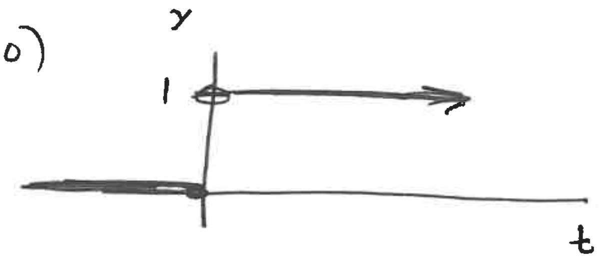
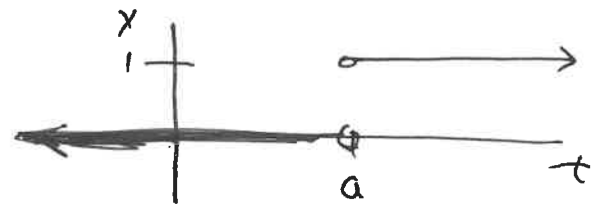
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

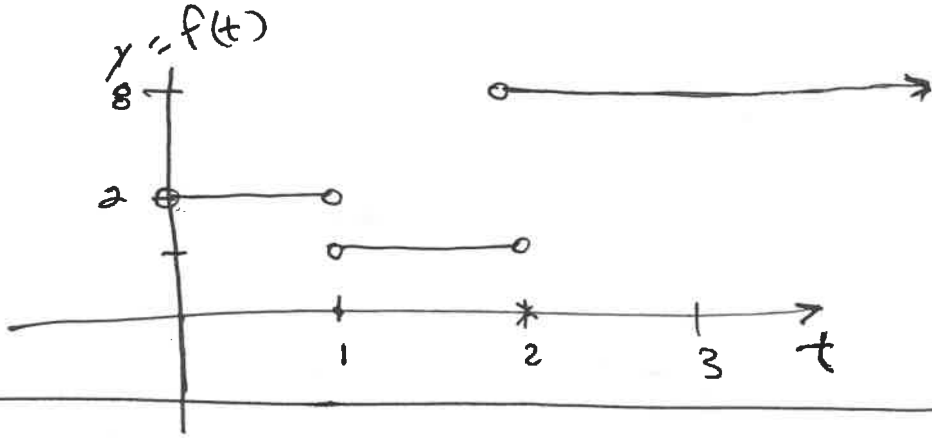
u(t - 0)



$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$



ex.)

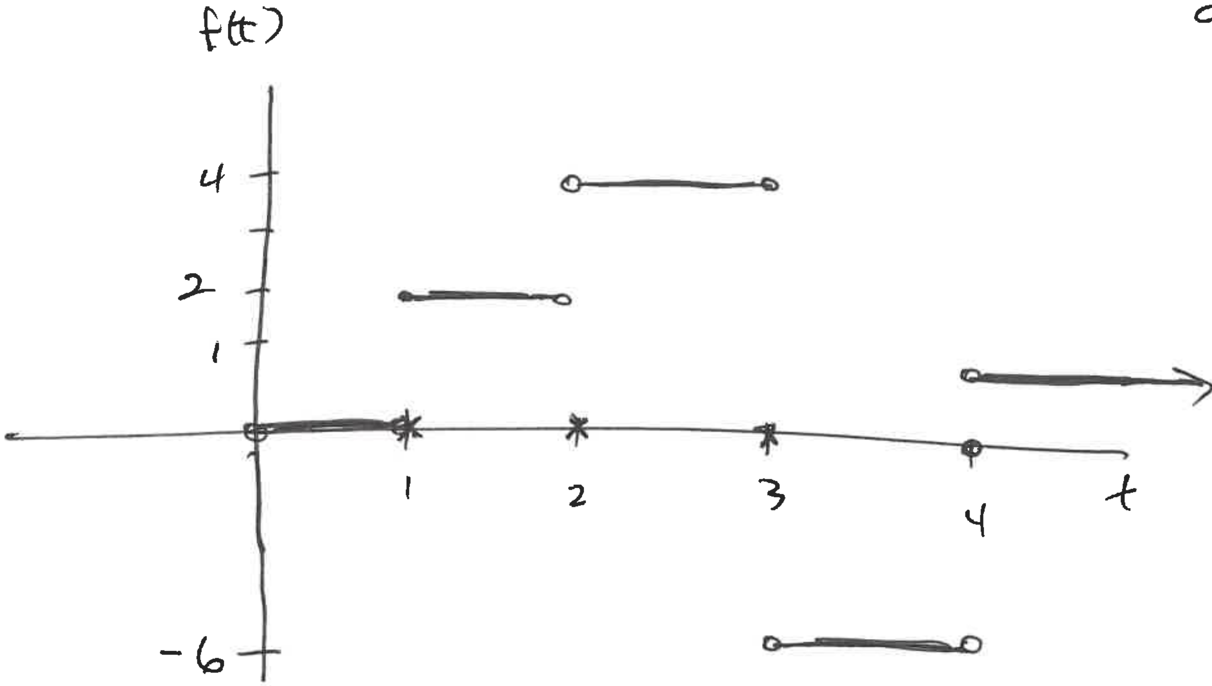


$$\boxed{f(t) = 2\left[u(t) - u(t-1)\right] + 1\left[u(t-1) - u(t-2)\right] + 8\,u(t-2)}$$

$$= 2 - 2u(t-1) + u(t-1) - u(t-2) + 8u(t-2)$$

$$* \; f(t) = 2 - u(t-1) + 7u(t-2)$$

ex.)



$$f(t) = \cancel{0\left[u(t) - u(t-1)\right]} + 2\left[u(t-1) - u(t-2)\right] + 4\left[u(t-2) - u(t-3)\right]$$
$$- 6\left[u(t-3) - u(t-4)\right] + 1\,u(t-4)$$

$$= 2u(t-1) - 2u(t-2) + 4u(t-2) - 4u(t-3) - 6u(t-3)$$
$$+ 6u(t-4) + u(t-4)$$

$$f(t) = 2u(t-1) + 2u(t-2) - 10u(t-3) + 7u(t-4)$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s} \qquad * \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{f(t)\} = 2\mathcal{L}\{u(t-1)\} + 2\mathcal{L}\{u(t-2)\} - 10\mathcal{L}\{u(t-3)\}$$
$$+ 7\mathcal{L}\{u(t-4)\}$$

$$= \frac{2e^{-s}}{s} + \frac{2e^{-2s}}{s} - 10\frac{e^{-3s}}{s} + 7\frac{e^{-4s}}{s}$$

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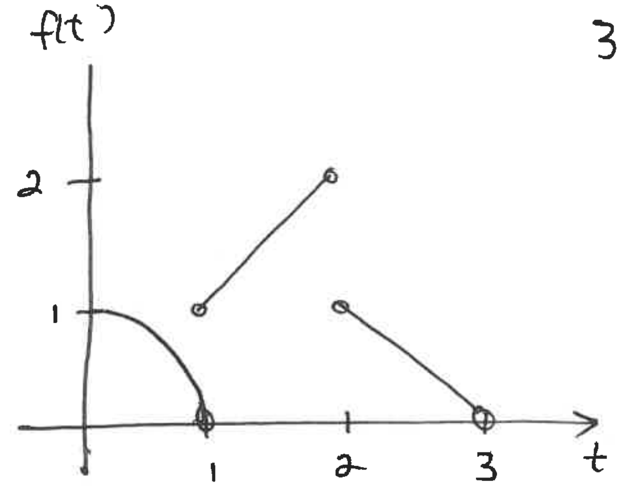
ex.) $f(t) = \begin{cases} 1-t^2, & 0<t<1 \\ t, & 1<t<2 \\ -t+3, & 2<t<3 \end{cases}$



$$f(t) = (1-t^2)\left[u(t) - u(t-1)\right] + t\left[u(t-1) - u(t-2)\right] + (3-t)\left[u(t-2) - u(t-3)\right]$$

$$= 1-t^2 - (1-t^2)u(t-1) + t\,u(t-1) - t\,u(t-2) + (3-t)\,u(t-2) + (t-3)u(t-3)$$

$$f(t) = 1-t^2 + t^2u(t-1) + (t-1)u(t-1) + (3-2t)\,u(t-2) + (t-3)u(t-3)$$

No spse we want to find $\mathcal{L}\{f(t)\}$.

$$\mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{2}{s^3} + \mathcal{L}\left\{(t+1)^2\right\}e^{-s} + e^{-s}\mathcal{L}\{t\} + e^{-2s}\mathcal{L}\left\{3-2(t+2)\right\} + e^{-3s}\mathcal{L}\{t\}$$

$$= \frac{1}{s} - \frac{2}{s^3} + e^{-s}\mathcal{L}\left\{t^2+2t+1\right\} + \frac{e^{-s}}{s^2} + e^{-2s}\mathcal{L}\left\{-1-2t\right\} + \frac{e^{-3s}}{s^2}$$

$$= \frac{1}{s} - \frac{2}{s^3} + e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) + \frac{e^{-s}}{s^2} + e^{-2s}\left(\frac{-1}{s} - \frac{2}{s^2}\right) + \frac{e^{-3s}}{s^2}$$

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#11 $\mathcal{L}^{-1}\left\{\dfrac{7e^{-2s}}{s-1}\right\} = 7e^{t-2} \quad u(t-2)$

#12 $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{s^2}\right\} = (t-3) \quad u(t-3)$

#14 $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{s^2+9}\right\} = \frac{1}{3}\sin(3(t-3)) \quad u(t-3)$

$\left(\frac{3}{3}\right)\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+9}\right\}$

$\frac{1}{3}\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\}$

$\frac{1}{3}\sin(3t)$

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#18 $\dfrac{e^{-s}(3s^2-s+2)}{(s-1)(s^2+1)}$

$$\frac{3s^2-s+2}{(s-1)(s^2+1)} = \frac{\overset{2}{A}}{s-1} + \frac{\overset{1}{B}s}{s^2+1} + \frac{\overset{\phi}{C}}{s^2+1}$$

Let $s=0$ $\quad \frac{2}{-1} = -2 + C$

$$0 = C$$

Let $s=2$ $\quad \frac{12}{5} = 2 + \frac{2B}{5}$

$$12 = 10 + 2B$$

$$1 = B$$

$$\left(2e^{t-1} + \cos(t-1)\right)u(t-1)$$

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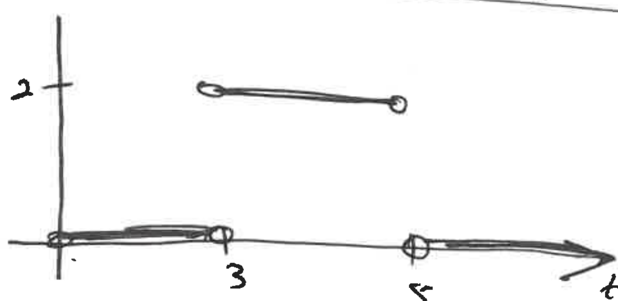
Solve $y'' + y' - 6y = \begin{cases} 0, & t < 3 \\ 2, & 3 < t < 5 \\ 0, & t > 5 \end{cases}$ $\quad y(0)=1$, $y'(0)=1$

LEFT

$$\left(s^2Y(s) - s\,\overset{1}{y(0)} - \overset{1}{y'(0)}\right) + \left(sY(s) - \overset{1}{y(0)}\right) - 6Y(s)$$

$$\boxed{(s^2+s-6)Y(s) - s - 2 = 2\left(\frac{e^{-3s}}{s} - \frac{e^{-5s}}{s}\right)}$$

Right Side



$$\cancel{0\left[u(t) - u(t-3)\right]} + 2\left[u(t-3) - u(t-5)\right] + \cancel{0\left[u(t-5)\right]}$$

$$Y(s) = \frac{2(e^{-3s} - e^{-5s})}{s(s+3)(s-2)} + \frac{s+2}{(s+3)(s-2)}$$

stepped

$$\frac{2\left(e^{-3s} - e^{-5s}\right)}{s(s+3)(s-2)}$$

$$\frac{2}{s(s+3)(s-2)} = \frac{\overset{-\frac{1}{3}}{A}}{s} + \frac{\overset{\frac{2}{15}}{B}}{s+3} + \frac{\overset{\frac{1}{5}}{C}}{s-2}$$

$$\left(-\frac{1}{3} + \frac{2}{15}e^{-3(t-3)} + \frac{1}{5}e^{2(t-3)}\right)u(t-3) -$$

$$\left(-\frac{1}{3} + \frac{2}{15}e^{-3(t-5)} + \frac{1}{5}e^{2(t-5)}\right)u(t-5)$$

Normal

$$\frac{\overset{-1}{s+2}}{(s+3)(s-2)} = \frac{\overset{\frac{1}{5}}{A}}{s+3} + \frac{\overset{\frac{4}{5}}{B}}{s-2}$$

$$y(t) = \frac{1}{5}e^{-3t} + \frac{4}{5}e^{+2t} + \left(-\frac{1}{3} + \frac{2}{15}e^{-3(t-3)} + \frac{1}{5}e^{2(t-3)}\right)u(t-3)$$

$$- \left(-\frac{1}{3} + \frac{2}{15}e^{-3(t-5)} + \frac{1}{5}e^{2(t-5)}\right)u(t-5)$$

cool