Section 2.2 6/12 1

#15 $\overset{N}{y^{-1}}\,dy + \overset{M}{\underline{y e^{\cos y} \sin x}}\, dx = 0$

check if exact $\dfrac{\partial M}{\partial y} = \text{Awful} \neq \dfrac{\partial N}{\partial x} = 0$

Nope!

$\frac{1}{y}\,dy = -y e^{\cos x} \sin x \, dx \Rightarrow \frac{dy}{dx} = (-y^2) e^{\cos x} \sin x$

sep!

$-\int \frac{1}{y^2}\,dy = \int e^{\cos x} \sin x \, dx$

$-\int y^{-2}\,dy = \int e^{\cos x} \sin x \, dx$ $\qquad -\int e^u \, du = -e^u$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$\frac{1}{y} + C_2 = -e^{\cos x} + C_1$

$\frac{1}{y} = -e^{\cos x} + C_3,$ $\qquad C_3 = C_1 - C_2$

$\therefore \boxed{y = \dfrac{1}{C_3 - e^{\cos x}}}$

☺

Pg 51 2.3

#16 $(1-x^2)\frac{dy}{dx} - x^2 y = (1+x)\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{yx^2 + (1+x)\sqrt{1-x^2}}{1-x^2}$$

$$\frac{-x^2}{1-x^2}$$

Must be linear (I hope)

$$\frac{dy}{dx} = \frac{x^2}{1-x^2}y + \frac{(1+x)\sqrt{1-x^2}}{1-x^2}$$

$$\frac{dy}{dx} + \frac{x^2}{x^2-1}y = \frac{(1+x)\sqrt{1-x^2}}{1-x^2}$$

$$\mu(x) = e^{\int \frac{x^2}{x^2-1}dx}$$

$$\int \frac{x^2}{x^2-1}dx$$

$$\int \frac{x^2}{x^2-1}\,dx = \int\left(1+\frac{1}{x^2-1}\right)dx = x + \int \frac{1}{x^2-1}\,dx$$

$$\begin{array}{r|l} & 1 \\ x^2-1 & x^2 \\ & \underline{x^2-1} \\ & 1 \end{array}$$

$$\frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1}$$

$$\frac{1}{(x+1)(x-1)} = \frac{x-1}{x-1}\frac{\overset{-\frac{1}{2}}{A}}{x+1} + \frac{\overset{\frac{1}{2}}{B}}{x-1}\frac{x+1}{x+1} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

$$(A+B)x + (-A+B) = 1$$

$$\begin{array}{r} A + B = 0 \\ -A + B = 1 \\ \hline 2B = 1 \\ B = \frac{1}{2} \end{array}$$

$$x + \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1|$$

$$\boxed{x + \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)} = \underline{\underline{x + \ln\sqrt{\frac{x-1}{x+1}}}}$$

$$\mu(x) = e^{x + \ln\sqrt{\frac{x-1}{x+1}}} = e^{x} \cdot e^{\ln\sqrt{\frac{x-1}{x+1}}}$$

$$\boxed{\mu(x) = e^{x}\sqrt{\frac{x-1}{x+1}}}$$

$$\sqrt{1-x^2} = \sqrt{-(x^2-1)}$$

$$\frac{dy}{dx} + \frac{x^2}{x^2-1}\,y = \frac{(1+x)\sqrt{1-x^2}}{1-x^2}$$

$$\frac{d}{dx}\left(e^{x}\sqrt{\frac{x-1}{x+1}}\,y\right) = \frac{(1+x)\sqrt{1-x^2}}{1-x^2}\,e^{x}\sqrt{\frac{x-1}{x+1}}$$

$$= e^{x}\,\frac{\cancel{(1+x)}\sqrt{1-x^2}}{\cancel{(1+x)}(1-x)}\,\frac{\sqrt{x-1}}{\sqrt{x+1}}$$

Miracle $\longrightarrow$ $$= e^{x}\,\frac{\sqrt{-\cancel{(x+1)}(x-1)}}{1-x}\,\frac{\sqrt{x-1}}{\cancel{\sqrt{x+1}}}$$

$$= e^{x}\,\frac{\sqrt{1-x}}{\sqrt{1-x}} = e^{x}$$

$$\frac{d}{dx}\left(e^{x}\sqrt{\frac{x-1}{x+1}}\,y\right) = e^{x}$$

$$e^{x}\sqrt{\frac{x-1}{x+1}}\,y = \int e^{x}\,dx = e^{x} + C$$

$$e^x \sqrt{\frac{x-1}{x+1}}\, y = e^x + C$$

$$y = \frac{1}{\sqrt{\frac{x-1}{x+1}}} + \frac{C}{e^x \sqrt{\frac{x-1}{x+1}}}$$

$$y = \sqrt{\frac{x+1}{x-1}} + \frac{C}{e^x \sqrt{\frac{x-1}{x+1}}}$$

Pg 61 Section 2.4

#7 $\left[2x + \overset{M}{y\cos(xy)}\right]dx + \left[\overset{N}{x\cos(xy) - 2y}\right]dy = 0$

$$\frac{\partial M}{\partial y} = y\left[-x\sin(xy)\right] + \cos(xy)$$

$$\parallel$$

$$\frac{\partial N}{\partial x} = x\left[-y\sin(xy)\right] + \cos(xy)$$

EXACT !

$$F(x,y) = \int (2x + y\cos(xy))\,dx = \int (2x)\,dx + \int y\cos(xy)\,dx$$

$$\boxed{F(x,y) = x^2 + \sin(xy) + \phi(y)}$$

$$\int y\cos(xy)\,dx = y\int \cos(xy)\,dx = y \cdot \frac{1}{y}\int \cos u\,du$$

$u = xy$

$du = y\,dx$

$\frac{1}{y}du = dx$

$$= \int \cos u\,du = \sin u$$

Now, we know $\dfrac{\partial F}{\partial y} = N$

$$x\cos(xy) + \phi'(y) = x\cos(xy) - 2y$$

$$\phi'(y) = -2y \Rightarrow \phi(y) = -y^2$$

$$\boxed{F(x,y) = x^2 + \sin(xy) - y^2 = C}$$

2.5 Special IF

#8 $\overset{M}{(3x^2+y)}\,dx + \overset{N}{(x^2y-x)}\,dy = 0$

$$\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = 2xy-1$$

$$* \quad \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \text{ is a fcn. of } x \text{ alone}$$

$$\frac{1-(2xy-1)}{x^2y-x} = \frac{1-2xy+1}{x^2y-x} = \frac{2-2xy}{x(xy-1)}$$

$$\frac{2\cancel{(1-xy)}}{-x\cancel{(1-xy)}} = -\frac{2}{x} \qquad = \frac{2(1-xy)}{x(xy-1)}$$

$$\mu(x) = e^{-2\int \frac{1}{x}\,dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$$

$$\frac{1}{x^2}\left[(3x^2+y)\,dx + (x^2y-x)\,dy = 0\right]$$

$$\overset{M}{(3+yx^{-2})}\,dx + \overset{N}{(y-x^{-1})}\,dy = 0$$

$$\frac{\partial M}{\partial y} = x^{-2} = \frac{\partial N}{\partial x} \quad \text{So now exact!}$$

#12 cont. 2.6 pg 74

So we have $\frac{1}{3} \ln|1+3v^2| + C_2 = -\ln|x| + C_1$

$$\ln|1+3v^2| + C_2 = -3\ln|x| + C_1$$

$$\ln|1+3v^2| + C_2 = \ln\left|\frac{1}{x^3}\right| + C_1$$

(here $C_2 = 3C$
$C_1 = 3C$)

$$\ln|1+3v^2| = \ln\left|\frac{1}{x^3}\right| + C_3, \quad C_3 = C_1 - C_2$$

$$|1+3v^2| = \frac{1}{x^3} \cdot e^{C_3}$$

$$|1+3v^2| = C_4 \frac{1}{x^3}, \quad C_4 = e^{C_3}$$

$$1+3v^2 = \pm C_4 \frac{1}{x^3} = C_5 \frac{1}{x^3}, \quad C_5 = \pm C_4$$

$$3v^2 = \frac{C_5}{x^3} - 1$$

$$v^2 = \frac{C_6}{x^3} - \frac{1}{3}, \quad C_6 = C_5/3$$

$$\left(\frac{y}{x}\right)^2 = v^2 = \frac{C_7 - x^3}{3x^3}, \quad C_7 = 3C_6$$

$$\boxed{y^2 = \frac{C_7 - x^3}{3x}}$$

Pg 74 2.6

#12 $(x^2+y^2)\,dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2+y^2)\,dx$$

$$\frac{dy}{dx} = -\frac{(x^2+y^2)}{2xy} = -\frac{1}{2}\frac{x^2}{xy} - \frac{1}{2}\frac{y^2}{xy}$$

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{x}{y}\right) - \frac{1}{2}\left(\frac{y}{x}\right)$$

Let $v = \frac{y}{x} \Rightarrow y = xv$ Then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = -\frac{1}{2}\cdot\frac{1}{v} - \frac{1}{2}v$$

$$x\frac{dv}{dx} = -\frac{1}{2}\cdot\frac{1}{v} - \frac{3}{2}v = \frac{-1}{2v} - \frac{3v}{2} = -\left(\frac{1+3v^2}{2v}\right)$$

Seperable

$$\frac{2v}{1+3v^2}\,dv = -\frac{1}{x}\,dx \Rightarrow 2\int\frac{v}{1+3v^2}\,dv = -\int\frac{1}{x}\,dx$$

$$= -\ln|x| + C_1$$

for $2\int\frac{v}{1+3v^2}\,dv = \frac{1}{3}\int\frac{1}{u}\,du$

Let $u = 1+3v^2$ $\quad = \frac{1}{3}\ln|u| + C_2$

$du = 6v\,dv$ $\quad = \frac{1}{3}\ln|1+3v^2| + C_2$

$\frac{1}{6}du = v\,dv$

2.6 pg 74 Bernoulli

#22

$$\frac{dy}{dx} - y = e^{2x} y^3$$

$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{y^2} = e^{2x}$$

let $\boxed{\begin{aligned} v &= y^{-2} \\ \frac{dv}{dx} &= -2y^{-3}\frac{dy}{dx} \\ -\frac{1}{2}\frac{dv}{dx} &= y^{-3}\frac{dy}{dx} \end{aligned}}$

$$\therefore -\frac{1}{2}\frac{dv}{dx} - v = e^{2x}$$

$$\frac{dv}{dx} + 2v = -2e^{2x}$$

I.F. $\mu(x) = e^{2\int dx} = e^{2x} \Rightarrow e^{2x}\frac{dv}{dx} + 2e^{2x}v = -2e^{4x}$

$$\frac{d}{dx}\left(e^{2x}v\right) = -2e^{4x}$$

Integrating $e^{2x}v = -\frac{1}{2}e^{4x} + c$

$$v = -\frac{1}{2}e^{2x} + \frac{c}{e^{2x}} = \frac{c}{e^{2x}} - \frac{e^{2x}}{2}$$

$$v = \frac{c_1 - e^{4x}}{2e^{2x}}, \quad c_1 = 2c$$

but $v = \frac{1}{y^2} \Rightarrow \frac{1}{y^2} = \frac{c_1 - e^{4x}}{2e^{2x}}$

$$y^2 = \frac{2e^{2x}}{c_1 - e^{4x}}$$