6/16/2017

Recap $\quad ay'' + by' + cy = 0$

Leads to Aux. eqn $\quad ar^2 + br + c = 0 \quad$ (soln. $\underline{y = e^{rt}}$)

1.) roots are real & distinct
2.) Double root
3.) roots $\mathbb{C}$.

Gen soln will be two functions y_1 & y_2 as long as y_1 is "different" from y_2

$y_1 \neq K y_2 \qquad y_1$ & y_2 Linearly Indep.

If $\underline{y_1 = K y_2}$ then y_1 & y_2 are Linearly Dep.

Play

$y_1 = -\frac{c_2}{c_1} y_2 \quad \Rightarrow \quad c_1 y_1 = -c_2 y_2$

SAME

$\boxed{c_1 y_1 + c_2 y_2 = 0}$

Any two solns y_1, y_2 then $y = c_1 y_1 + c_2 y_2$ is also a soln.

If $\underline{y_1}$ & $\underline{y_2}$ are LI then $\boxed{y = c_1 y_1 + c_2 y_2}$ Gen Soln

Wronskian

$$W[y_1, y_2] = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2 \neq 0$$

y_1 & y_2 are Lin. Ind.

If y_1 & y_2 are Lin. Dep. $\Longrightarrow$ Then $W[y_1, y_2] = 0$

If $W[y_1, y_2] = 0$ suggests y_1 & y_2 are Lin. Dep.

You must find c_1 & c_2 st $\left\{ \begin{array}{l} c_1 y_1 + c_2 y_2 = 0 \\ \text{or} \\ y_1 = k y_2 \end{array} \right.$

ex) $y_1 = 2^x$ & $y_2 = 2^{x+3}$

$$W[2^x, 2^{x+3}] = \begin{vmatrix} 2^x & 2^{x+3} \\ 2^x \ln 2 & 2^{x+3} \ln 2 \end{vmatrix}$$

$$= 2^{x+x+3} \ln 2 - 2^{x+x+3} \ln 2 = 0$$

$$W[t^3, t^2] = \begin{vmatrix} t^3 & t^2 \\ 3t^2 & 2t \end{vmatrix} = 2t^4 - 3t^4 = -t^4 \neq 0$$

$\therefore y_1 = t^3$ & $y_2 = t^2$ are L.I.

$$y = c_1 t^3 + c_2 t^2$$

i.) r_1 & r_2 & $r_1 \neq r_2 \in \mathbb{R}$ $\quad y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

* $\left\{ e^{r_1 t}, e^{r_2 t} \right\}$ form a fund. Soln. Set.

fund. Soln Set are the Linearly Indep. solns to the homogenous eqn.

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$$2y'' + 7y' - 4y = 0$$

$\therefore$ Aux EQN $\quad 2r^2 + 7r - 4 = 0$

$$(2r-1)(r+4) = 0$$

$\therefore r_1 = \frac{1}{2} \quad r_2 = -4 \implies e^{\frac{1}{2}t}$ & e^{-4t}

$\underline{\left\{ e^{\frac{1}{2}t}, e^{-4t} \right\}} \implies y = c_1 e^{\frac{1}{2}t} + c_2 e^{-4t}$

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\# 9 $4y'' - 4y' + y = 0 \Rightarrow$ AUX. EQN

$$4r^2 - 4r + 1 = 0$$

$$(2r-1)^2 = 0$$

$$r_1 = \tfrac{1}{2}, \tfrac{1}{2}$$

$* \left\{ e^{\frac{1}{2}t}, t e^{\frac{1}{2}t} \right\}$

Soln $y = C_1 e^{\frac{1}{2}t} + C_2 t e^{\frac{1}{2}t}$

* Spse roots to AUX are $\mathbb{C}$

$r_1 = \alpha + i\beta$ $\qquad r_2 = \alpha - i\beta$

$e^{(\alpha+i\beta)t}$ $\qquad e^{(\alpha-i\beta)t}$

$$e^{\alpha t + i\beta t} = e^{\alpha t} \cdot \boxed{e^{i\beta t}}$$

Euler Formula $\qquad e^{i\theta} = \cos(\theta) + i\sin(\theta)$ *

If $r_1 = \alpha + i\beta$

$$\begin{cases} e^{\alpha t}\left(\cos(\beta t) + i\sin(\beta t)\right) & \text{is a soln} \\ e^{\alpha t}\left(\cos(\beta t) - i\sin(\beta t)\right) & \text{is a soln,} \end{cases}$$

$e^{\alpha t}\cos(\beta t) + i e^{\alpha t}\sin(\beta t)$. $\qquad r = \alpha + i\beta$

$e^{\alpha t}\cos(\beta t) - i e^{\alpha t}\sin(\beta t)$. $\qquad r = \alpha - i\beta$

$2 e^{\alpha t}\cos(\beta t) \Rightarrow \quad y_1 = c_1 e^{\alpha t}\cos(\beta t)$

Subtract $\quad 2 i e^{\alpha t}\sin(\beta t)$ must be a soln

$y_2 = c_2 e^{\alpha t}\sin(\beta t)$

I now have $\quad y_1 = c_1 e^{\alpha t}\cos(\beta t)$

$y_2 = c_2 e^{\alpha t}\sin(\beta t)$ as solns.

$$\begin{vmatrix} e^{\alpha t}\cos(\beta t) & e^{\alpha t}\sin(\beta t) \\ -\beta e^{\alpha t}\sin(\beta t) + \alpha e^{\alpha t}\cos(\beta t) & \beta e^{\alpha t}\cos(\beta t) + \alpha e^{\alpha t}\sin(\beta t) \end{vmatrix}$$

$e^{\alpha t} \neq 0$

$ay'' + by' + Cy = 0$ AUX

$ar^2 + br + C = 0$

i.) $r_1 \neq r_2$ $\{e^{r_1 t}, e^{r_2 t}\} \Rightarrow y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

ii.) Double root $\{e^{rt}, te^{rt}\} \Rightarrow y = C_1 e^{rt} + C_2 te^{rt}$

iii.) Root $\mathbb{C}$

$r = \alpha \pm i\beta$ $\{e^{\alpha t}\cos(\beta t), e^{\alpha t}\sin(\beta t)\}$

$y = C_1 e^{\alpha t}\cos(\beta t) + C_2 e^{\alpha t}\sin(\beta t)$

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#12 $u'' + 7u = 0 \Rightarrow$ AUX EQN

$r^2 + 7 = 0$ $\qquad r = \pm\sqrt{-7}$

$\{\cos(\sqrt{7}t), \sin(\sqrt{7}t)\}$ $\qquad r = \pm i\sqrt{7}$ $\qquad \pm\sqrt{-1}\cdot\sqrt{7}$

$0 \pm i\sqrt{7}$

$y = C_1\cos(\sqrt{7}t) + C_2\sin(\sqrt{7}t)$

#17 $y'' - y' + 7y = 0$ $\qquad$ Aux : $r^2 - r + 7 = 0$

FSS $\qquad r = \dfrac{1 \pm \sqrt{1-28}}{2}$

$\left\{e^{\frac{t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right), e^{\frac{t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)\right\}$ $\qquad r = \dfrac{1 \pm i\,3\sqrt{3}}{2} = \underset{\alpha}{\frac{1}{2}} \pm \underset{\beta}{\frac{3\sqrt{3}}{2}}i$

$y = C_1 e^{\frac{t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right) + C_2 e^{\frac{t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)$

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17.) $y'' - 2y' + y = 8e^{t}$

i) Solve $y'' - 2y' + y = 0$

Aux: $r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r = 1, r = 1$

$\{e^{t}, te^{t}\}$

$y_h = C_1 e^{t} + C_2 te^{t}$

Gen. Soln $y = y_h + y_p$

Guess $y_p = At^2 e^{t}$

y_p'

y_p''