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Ch. 4

4.2 Begin $a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$

$a \neq 0$ homogeneous

what could be the soln. or what kind of a fun. will work?

Guess $y(t) = e^{rt}$ (BTW a very good one)

$y'(t) = re^{rt}$

$y''(t) = r^2e^{rt}$

Now Plug & Chug

$$a(r^2e^{rt}) + b(re^{rt}) + ce^{rt} = 0$$

$$e^{rt}(ar^2 + br + c) = 0 \quad \therefore$$

either $\cancel{e^{rt} = 0}$ or $\underline{ar^2 + br + c = 0}$ Aux. EQN char. EQN

Know $e^{rt} > 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \begin{matrix} r_1 \\ r_2 \end{matrix}$$

i.) $\mathbb{R}$ $r_1 \neq r_2$ (roots unique) BTW $\Rightarrow b^2 - 4ac > 0$

ii.) $r_1 = r_2$ (double root) $\Rightarrow b^2 - 4ac = 0 \Rightarrow r = \frac{-b}{2a}$

iii.) $r_1, r_2 \in \mathbb{C}$ $r_1 = \alpha + \beta i$, $r_2 = \alpha - \beta i$ $\Rightarrow b^2 - 4ac < 0$

ex.)

#4 pg 165 $\quad y'' + 5y' + 6y = 0$

Given Soln is $\boxed{y(t) = e^{r_i t}}$

<u>Aux EQN</u>: $\iff r^2 + 5r + 6 = 0$

$(r+2)(r+3) = 0 \implies r_1 = -2$, $r_2 = -3$

$\therefore$ Given $y_1(t) = e^{-2t}$ & $y_2(t) = e^{-3t}$

$y_1 = K y_2$

Questions:

* 1) Just how many answers are there!

easy 2.) Know (<u>hope</u>) $y_1 = e^{-2t}$ & $y_2 = e^{-3t}$ are solns

ok 3.) Is there a <u>general</u> solution?

#2 check $\quad y_1 = e^{-2t}$

$y_1' = -2e^{-2t} \qquad y_1'' = 4e^{-2t}$

$$4e^{-2t} + 5(-2e^{-2t}) + 6e^{-2t} = e^{-2t}(4 - 10 + 6) = 0$$

yes $y_1 = e^{-2t}$ is a solution.

it is the case (you check) $y_2 = e^{-3t}$ is a soln.

we have $y'' + 5y' + 6y = 0$

& have found $y_1 = e^{-2t}$ & $y_2 = e^{-3t}$

are solutions

Question: Is $\boxed{y = C_1 e^{-2t} + C_2 e^{-3t}}$ itself a solution? Here $C_1, C_2 \in \mathbb{R}$

$$y = C_1 e^{-2t} + C_2 e^{-3t}$$

$$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t}$$

$$y'' = 4C_1 e^{-2t} + 9C_2 e^{-3t}$$

$$\left[4C_1 e^{-2t} + 9C_2 e^{-3t}\right] + 5\left[-2C_1 e^{-2t} - 3C_2 e^{-3t}\right] + 6\left[C_1 e^{-2t} + C_2 e^{-3t}\right] \stackrel{?}{=} 0$$

$$e^{-2t}\left(\underbrace{4C_1 - 10C_1 + 6C_1}_{\phi}\right) + e^{-3t}\left(\underbrace{9C_2 - 15C_2 + 6C_2}_{\phi}\right) \stackrel{?}{=} 0$$

So absolutely $y = C_1 e^{-2t} + C_2 e^{-3t}$ is a solution ☺

Linear Independence

Previous problem, found $\underline{y_1 = e^{-2t}}$ & $\underline{y_2 = e^{-3t}}$

Are they $\underline{\text{distinct}}$ $\Rightarrow$ $\underline{y_1 \neq k y_2}$

if distinct

ex.) $y_1 = 2^x$ $y_2 = 2^{x+3}$

$2^x = 2^{x+3} = 2^x \cdot 2^3$

y_1 y_2

$y_1 = \frac{y_2}{8}$ y_1

$$\boxed{\underline{c_1 y_1 + c_2 y_2 = 0}}$$

$c_1 y_1 = -c_2 y_2$

$y_1 = \boxed{\frac{-c_2}{c_1}} y_2 = K y_2$

Der. Discrim $ay'' + by' + cy = 0$ $y(t_0) = y_0$

$\underline{a y_1'' + b y_1' + c y_1 = y_0}$ $y'(t_0) = y_1$

$$3 \begin{cases} 5c_1 + 2c_2 = 0 \\ 3c_1 - 6c_2 = 9 \end{cases}$$

exp. (pg 162)

$*$ $y_1 y_2' - y_1' y_2 \neq 0$

DE $\quad ay'' + by' + cy = 0$

found two. Solns y_1 & y_2

Iff $\quad y_1 y_2' - y_1' y_2 \neq 0$ over some Inter.

then y_1 can Not be written as a mult. of y_2 (y_1 & y_2 are Linearly Indep)

Then if LI General Soln is

$$\underline{y = c_1 y_1 + C_2 y_2} \quad \star$$

clever: For y_1 & y_2 to be Linearly Indep.

$$\underline{y_1 y_2' - y_1' y_2} \neq 0$$

$$\Downarrow$$

$$w(y_1, y_2) \doteq \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2 \neq 0$$

$w(y_1, y_2)$ is the WROSKIAN

$$W\left[e^{-2t}, e^{-3t}\right] = \begin{vmatrix} e^{-2t} & e^{-3t} \\ -2e^{-2t} & -3e^{-3t} \end{vmatrix}$$

$$= -3e^{-5t} + 2e^{-5t} = -e^{-5t} < 0$$

Never zero

$\therefore$ e^{-2t} & e^{-3t} are <u>linearly Indep.</u>

means it is not possible to write $e^{-2t} = K e^{-3t}$

$\therefore$ Since LI $\quad y = C_1 e^{-2t} + C_2 e^{-3t}$

#6 pg 165 section 4.2

#6 $\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 16y = 0$

Guess $y = e^{rt}$ $\Rightarrow$ Aux eqn is

$r^2 + 8r + 16 = 0$

$(r+4)(r+4) = 0$

$y_1 = e^{-4t}$ $te^{-4t} = y_2$

$ar^2 + br + c = 0$

$r = \frac{-b \pm \sqrt{\cancel{b^2-4ac}}}{2a}$

double root $r = \frac{-b}{2a}$

$2ar + b = 0$

Let $y_1 = e^{-4t}$

$$\boxed{y_2 = te^{-4t}}$$

$$w[e^{-4t}, te^{-4t}] = \begin{vmatrix} e^{-4t} & te^{-4t} \\ -4e^{-4t} & -4te^{-4t} + e^{-4t} \end{vmatrix} =$$

$$-4te^{-8t} + e^{-8t} + 4te^{-4t} = e^{-8t} \neq 0$$

If in Aux. EQn. we have double Roots $y_1 = e^{rt}$ Then another soln that is linearly Indep.

$$y_2 = te^{rt}$$

$\therefore$ gen. soln $y = C_1 e^{rt} + C_2 t e^{rt}$

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