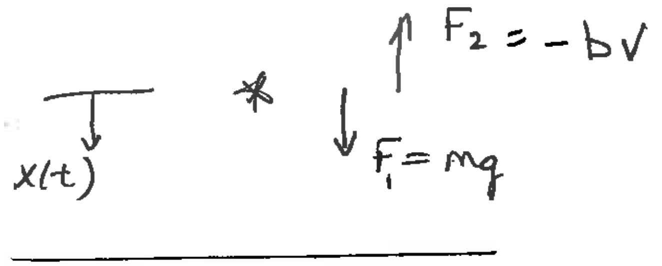
3.4 Example 1 (pg 110) (revisited)



we have

$$F_{Total} = ma = m\frac{dv}{dt}$$

$$m\frac{dv}{dt} = mg - bv \qquad \text{where} \quad v(0) = v_0 \quad \text{Init. vel.}$$

$$\frac{dv}{dt} = g - \frac{b}{m}v \implies \frac{dv}{dt} + \frac{b}{m}v = g$$

(Linear)

$$\mu(t) = e^{\frac{b}{m}\int dt} = e^{b/m\, t}$$

then

$$\frac{d}{dt}\left(e^{\frac{b}{m}t}v\right) = g e^{\frac{b}{m}t}$$

$$e^{\frac{b}{m}t}\, v = \frac{mg}{b} e^{\frac{b}{m}t} + C$$

$$v = \frac{mg}{b} + Ce^{-\frac{b}{m}t}$$

when $t = 0$, $v = v_0$ so $v_0 = \frac{mg}{b} + c$

$$v_0 - \frac{mg}{b} = c$$

$$V = \frac{mg}{b} + \left(V_0 - \frac{mg}{b}\right) e^{-\frac{b}{m}t} \quad *$$

Now $\quad \frac{dx}{dt} = V \Rightarrow x = \int v\,dt$

$$x = \int \left(\frac{mg}{b} + \left(V_0 - \frac{mg}{b}\right) e^{-\frac{b}{m}t} \right) dt$$

$$X = \frac{mg}{b} t - \frac{m}{b}\left(V_0 - \frac{mg}{b}\right) e^{-\frac{b}{m}t} + C_2$$

remember $x = x(t)$ is distance object has fallen.

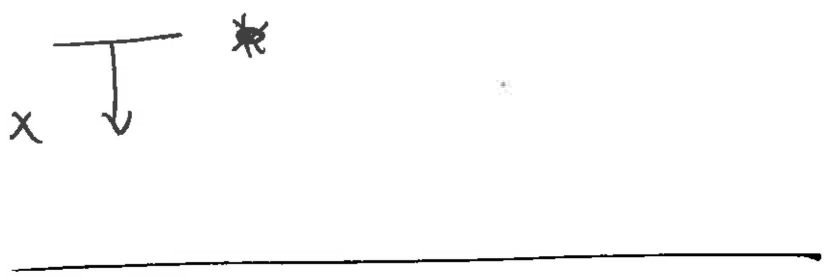
$\therefore \quad x(0) = 0 \quad$ Then

$$0 = -\frac{m}{b}\left(V_0 - \frac{mg}{b}\right) + C_2$$

$$C_2 = \frac{m}{b}\left(V_0 - \frac{mg}{b}\right)$$

So finally

$$x = \frac{mg}{b} t + \frac{m}{b}\left(V_0 - \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right)$$



3.4

#2

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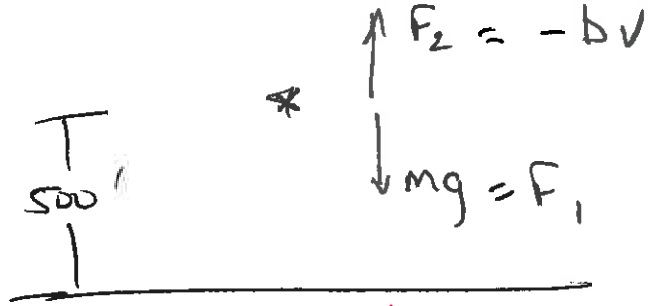


400 lbf

$mg = 400$

$g = 32.2$

$$m = \frac{400}{32.2} \Rightarrow \frac{m}{b} = \frac{400/32.2}{10}$$

$$X = \frac{mg}{b}t + \frac{m}{b}\left(V_0 - \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right)$$

given $F_2 = -10v \Rightarrow \boxed{b = +10}$

Also object is released $\Rightarrow V_0 \equiv 0$

$$X = \frac{mg}{b}t - \frac{m}{b}\left(\frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right)$$

$$X = 40t - \frac{40}{32.2}\left(\frac{\overset{40}{\cancel{400}}}{\cancel{10}}\right)\left(1 - e^{-\frac{322}{400}t}\right)$$

$$X = 40t - \frac{1600}{32.2}\left(1 - e^{-\frac{322}{400}t}\right)$$

$$X = 40t - 49.7\left(1 - e^{-0.805t}\right)$$

3.4

#2 when the object hits the ground

$x = 500$ so then

$$500 = 40t - 49.7\left(1 - e^{-0.805t}\right)$$

This gives $t \approx 13.7424805092 65$

Now $e^{-(0.805)(13.7)} \approx 0.0000162$

So if we ignore this term we have

$$40t = 549.7$$

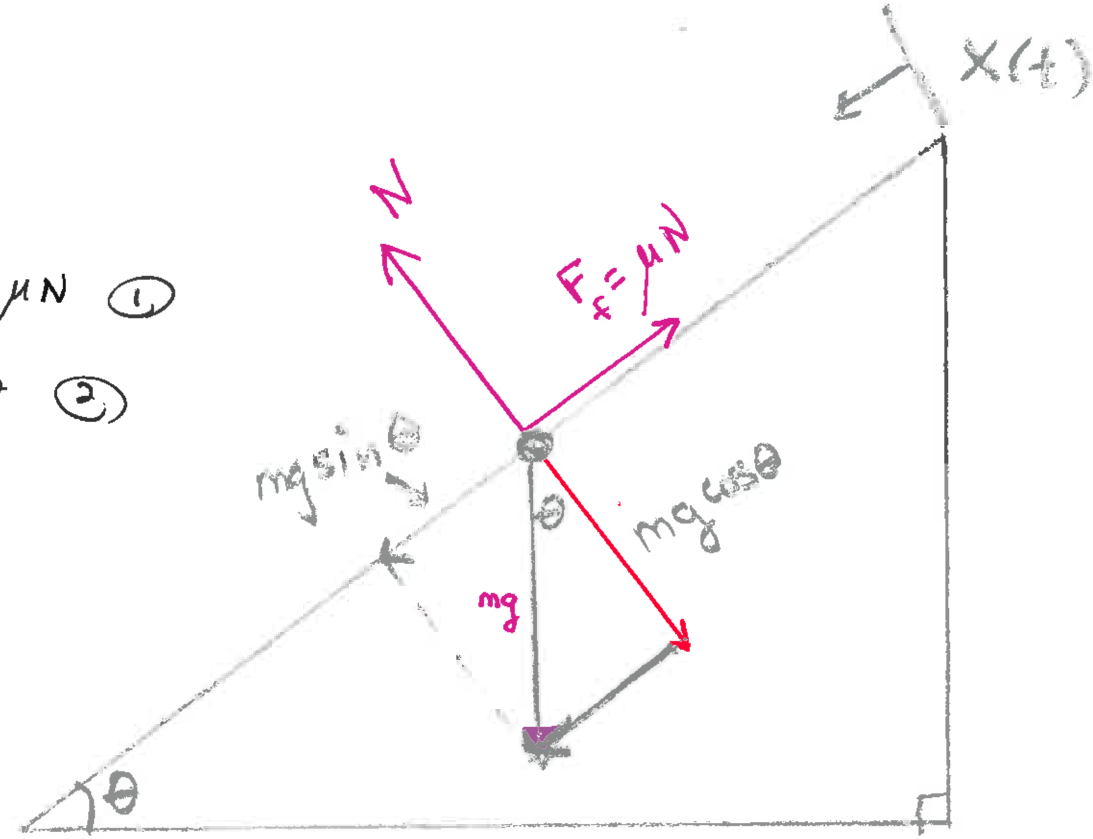
$$t = \frac{549.7}{40} \approx 13.7425$$

So object will hit ground after $t \approx \underline{13.7 \text{ sec.}}$

$x: \ \Sigma F = 0 = mg\sin\theta - \mu N$ (1)

$y: \ \Sigma F = 0 = N - mg\cos\theta$ (2)

$F = ma = m\frac{dv}{dt}$ (3)



From (2) $N = mg\cos\theta$ Then by (1) $mg\sin\theta - mg\cos\theta = 0$

by (3) & (1) $m\frac{dv}{dt} = mg(\sin\theta - \cos\theta)$ in the dir. of motion

$$m\frac{dv}{dt} = mg(\sin\theta - \mu\cos\theta) \implies \frac{dv}{dt} = \underbrace{g(\sin\theta - \mu\cos\theta)}_{\text{constant}}$$

$$dv = g(\sin\theta - \mu\cos\theta)\,dt$$

$$V = g(\sin\theta - \mu\cos\theta)\,t + C_1$$

when $t=0$, $V=0 \implies C_1 = 0$

$$\boxed{V = g(\sin\theta - \mu\cos\theta)\,t}$$

$$\frac{dx}{dt} = v \implies \int dx = x = \int v\,dt$$

$$x = \int g(\sin\theta - \mu\cos\theta)\,t\,dt$$

$$x = \frac{1}{2}g(\sin\theta - \mu\cos\theta)t^2 + C_2$$

when $t=0$, $x=0 \implies C_2 = 0$

$$\boxed{x = \frac{1}{2}g(\sin\theta - \mu\cos\theta)t^2}$$