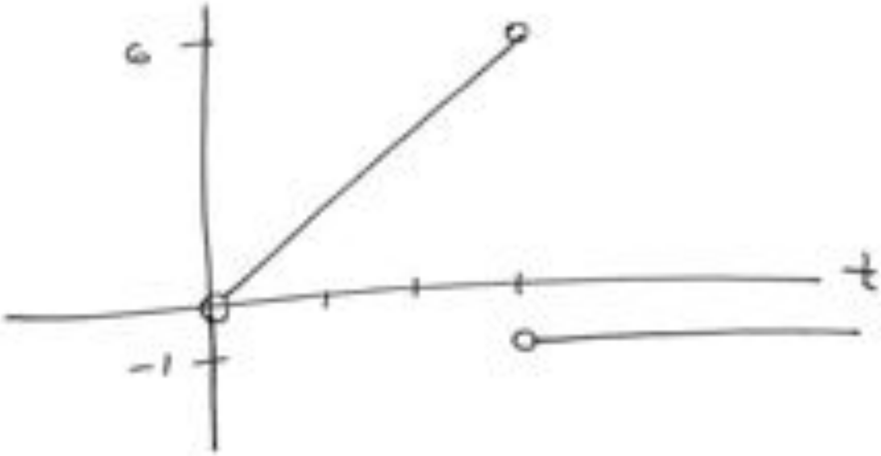
$\boxed{7.6}$

Recall $g(t) = \begin{cases} 2t, & 0 < t < 3 \\ -1, & t > 3 \end{cases}$

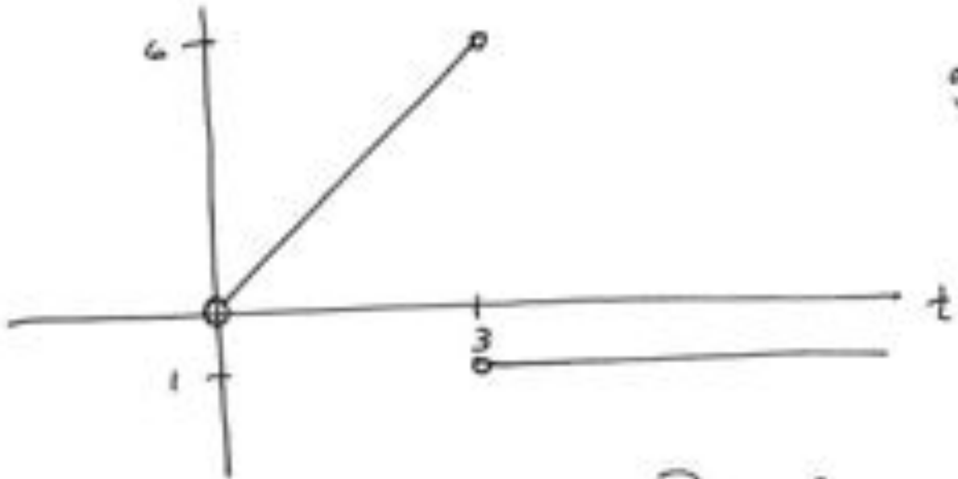


we found $\mathcal{L}\{g(t)\}$ using the formal definition

$$\mathcal{L}\{g(t)\} = \int_0^3 e^{-st}(2t)\,dt + \int_3^\infty e^{-st}(-1)\,dt$$

$$= \int_0^3 e^{-st}(2t)\,dt + \lim_{K\to\infty}\int_3^K e^{-st}(-1)\,dt$$

$$= \frac{2}{s^2} - \frac{2}{s^2}e^{-3s} - \frac{7}{s}e^{-3s}, \quad s > 0.$$



$$g(t) = \begin{cases} 2t, & 0<t<3 \\ -1, & t>3 \end{cases}$$

$$g(t) = 2t\left[u(t) - u(t-3)\right] - 1\left[u(t-3)\right]$$

$$g(t) = 2t\,u(t) - 2t\,u(t-3) - u(t-3)$$

$$g(t) = 2t\,u(t) - (2t-1)\,u(t-3)$$

Now, recall $\mathcal{L}\{u(t)\} = \frac{1}{s}$ & $\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$

and also we have $\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u(t-a)$.

ok then if $\mathcal{L}\{f(t)\} = F(s)$ then

$$\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}\,F(s)$$

so if we can match the argument of $f(t)$ to $u(t-a)$ — easy!

although $\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$

is nice - usually we end up with

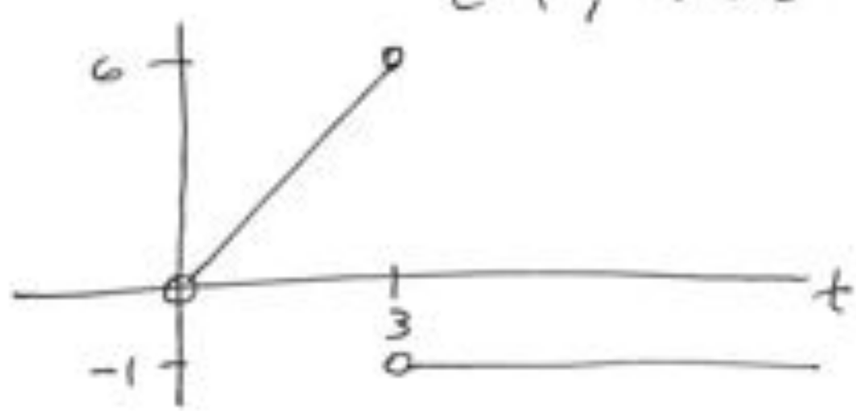
$\mathcal{L}\{g(t)\,u(t-a)\}$ The arguments do not match - so now what? Play a trick!!

$$g(t) = g((t+a)-a) \quad \text{so}$$

$$\mathcal{L}\{g(t)\,u(t-a)\} = \mathcal{L}\{g((t+a)-a)\,u(t-a)\} = e^{-as}\,\mathcal{L}\{g(t+a)\}$$

So when this happens take the original $g(t)$ then just replace t with $t+a$ and find the transform.

so remember $g(t) = \begin{cases} 2t, & 0<t<3 \\ -1, & t>3 \end{cases}$



$$g(t) = 2t\,[u(t) - u(t-3)] - u(t-3)$$

$$g(t) = 2t\,u(t) - (2t+1)\,u(t-3)$$

$$g(t) = 2t\,u(t) - (2t+1)\,u(t-3)$$

Then $\mathcal{L}\{g(t)\} = \mathcal{L}\{2t\} - \mathcal{L}\{(2t+1)\,u(t-3)\}$

$$= \frac{2}{s^2} - e^{-3s}\,\mathcal{L}\{2(t+3)+1\}$$

$$= \frac{2}{s^2} - e^{-3s}\,\mathcal{L}\{2t+7\}$$

$$= \frac{2}{s^2} - e^{-3s}\left(\frac{2}{s^2} + \frac{7}{s}\right)$$

$$= \frac{2}{s^2} - \frac{2}{s^2}e^{-3s} - \frac{7}{s}e^{-3s}$$

as before when we used the formal def!

ex.) $\mathcal{L}\{(\cos t)\, u(t-\pi)\} = \mathcal{L}\{\cos((t+\pi)-\pi)\, u(t-\pi)\}$

$= e^{-\pi s}\, \mathcal{L}\{\cos(t+\pi)\}$

now $\cos(t+\pi) = \cos t \cos\pi \cancel{- \sin t \sin\pi} = -\cos t$

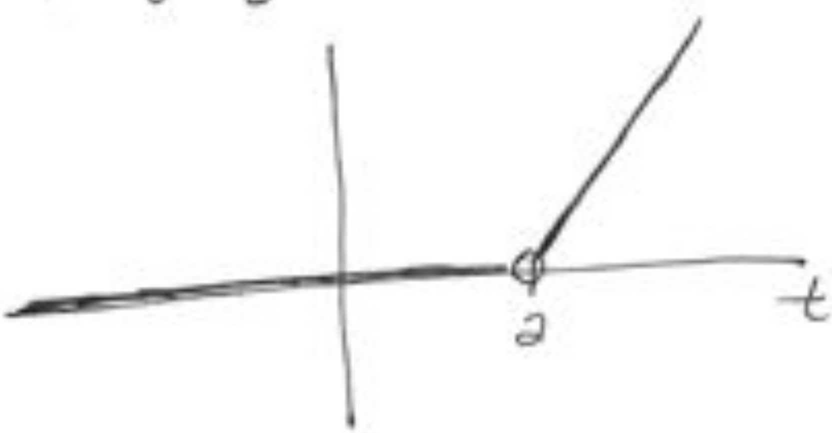
so $e^{-\pi s}\, \mathcal{L}\{\cos(t+\pi)\} = -e^{-\pi s}\, \mathcal{L}\{\cos t\}$

$\therefore \mathcal{L}\{(\cos t)\, u(t-\pi)\} = -e^{-\pi s} \dfrac{s}{s^2+1}$

ex.) $\mathcal{L}^{-1}\left\{ \dfrac{e^{-2s}}{s^2} \right\} = f(t-2)\, u(t-2) = \underline{(t-2)\, u(t-2)}$

write $\dfrac{e^{-2s}}{s^2} = e^{-2s}\left(\dfrac{1}{s^2}\right)$

and $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\} = t = f(t)$



Def: A function $f(t)$ is <u>periodic</u> with period T if $f(t+T) = f(t)$

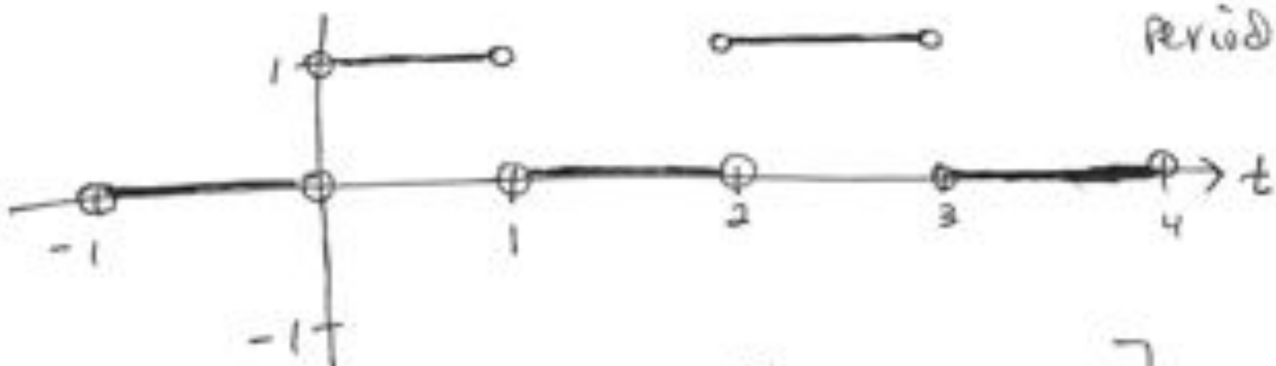
Denote $f_T(t)$ to be just one period (cycle) of the periodic function $f(t)$.

THM 9: If $f(t)$ has period T and is pw cont. on $[0,T]$. Then
pg 417

$$\mathcal{L}\{f(t)\} = \frac{F_T(s)}{1-e^{-sT}}$$

ex.)

period: 2



$$f_T(t) = 1\left[u(t) - u(t-1)\right] + 0\left[u(t-1) - u(t-2)\right]$$

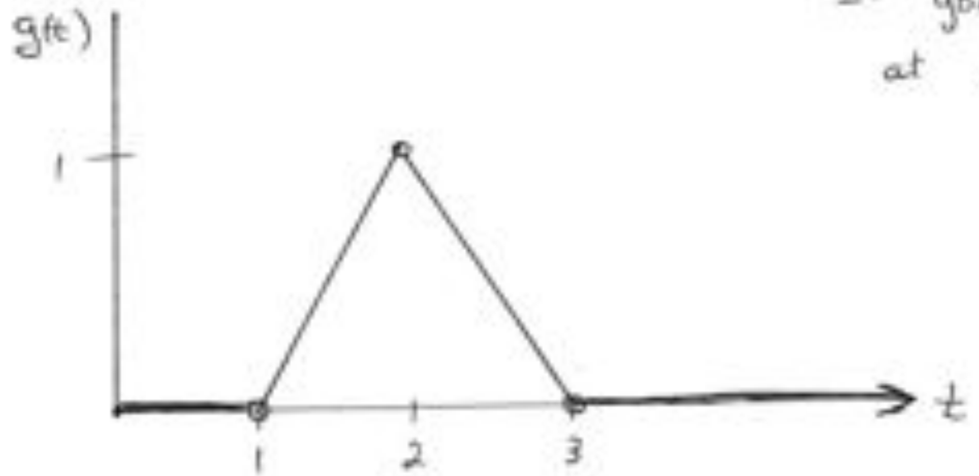
$$f_T(t) = u(t) - u(t-1) \quad \text{Then}$$

$$\mathcal{L}\{f_T(t)\} = \mathcal{L}\{u(t) - u(t-1)\} = \frac{1}{s} - \frac{e^{-s}}{s} \quad \text{so}$$

$$\mathcal{L}\{f(t)\} = \frac{\frac{1}{s}\left(1-e^{-s}\right)}{1-e^{-2s}} = \frac{\frac{1}{s}\left(1-e^{-s}\right)}{\left(1-e^{-s}\right)\left(1+e^{-s}\right)} = \frac{1}{s\left(1+e^{-s}\right)}$$

#9 pg 421

If you need to start at origin.



$$g(t) = 0\left[u(t) - u(t-1)\right] + (t-1)\left[u(t-1) - u(t-2)\right] + (3-t)\left[u(t-2) - u(t-3)\right]$$

$$= (t-1)u(t-1) + \Big((1-t) + (3-t)\Big)u(t-2) + (t-3)u(t-3)$$

$$g(t) = (t-1)u(t-1) + (4-2t)u(t-2) + (t-3)u(t-3)$$

now for $\mathcal{L}\{g(t)\}$ on right look at LT of each term.

$$\mathcal{L}\{(t-1)u(t-1)\} = e^{-s}\mathcal{L}\{t\} = e^{-s}\left(\frac{1}{s^2}\right)$$ here $f(t) = t$ then $f(t-1) = t-1$

$$\mathcal{L}\{(4-2t)u(t-2)\} = \mathcal{L}\{(\overbrace{4-2(t+2)}^{f(t-2)})u(t-2)\} = \mathcal{L}\{(-2t)u(t-2)\}$$

$$= e^{-2s}\left(\frac{-2}{s^2}\right)$$

$$\mathcal{L}\{(t-3)u(t-3)\} = e^{-3s}\mathcal{L}\{t\} = e^{-3s}\left(\frac{1}{s^2}\right)$$ again if $f(t-3) = t-3$ then $\underline{f(t) = t}$.

So $$\mathcal{L}\{g(t)\} = \left(e^{-s} - 2e^{-2s} + e^{-3s}\right)\left(\frac{1}{s^2}\right).$$

7.6 #13

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}-3e^{-4s}}{s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s+2}\right\} - 3\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t}$$

in first term the e^{-2s} shifts the t in e^{-2t} two units and "turns on"

$$\text{So } \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s+2}\right\} = e^{-2(t-2)}\,u(t-2)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+2}\right\} = e^{-2(t-4)}\,u(t-4)$$

so

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}-3e^{-4s}}{s+2}\right\} = e^{-2(t-2)}\,u(t-2) - 3e^{-2(t-4)}\,u(t-4).$$