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#34 $T \equiv$ temp. of obj. at time t

$M \equiv$ temp. of surroundings

$$\frac{dT}{dt} = K(M-T)$$

a.) Solve, note separable!

$$\frac{1}{M-T}\,dT = K\,dt$$

$$\int \frac{1}{M-T}\,dT = K\int dt = Kt + C_1$$

$$-\ln|M-T| = Kt + C_1$$

$$\ln|M-T| = -Kt - C_1$$

$$\ln|M-T| = -Kt + C_2, \qquad C_2 = -C_1$$

$$|M-T| = C_3 e^{-Kt}, \qquad C_3 = e^{C_2} > 0.$$

Now Note: $M - T \geq 0$, temp. of obj. cannot be colder than its surroundings.

$$M - T = C_3 e^{-kt} \Rightarrow \underline{T = T(t) = M - C_3 e^{-kt}}$$

if we let $T(0) = T_0$ Then

$$T_0 = M - C_3 \quad \text{OR} \quad C_3 = M - T_0$$

So $$T = M - (M - T_0) e^{-kt} \quad \text{OR}$$

$$\boxed{T = M + (T_0 - M) e^{-Kt}}$$

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34 continued

b) Thermometer where $T_0 = 100°F$

$M = 70°F$ & $T(6) = 80°F$

what is $T(20)$?

Using the info:

$$T = 70 + (100-70)e^{-Kt}$$

$$T = 70 + 30e^{-Kt} \quad * \text{ (see note at end)}$$

If $T(6) = 80$ Then

$$80 = 70 + 30e^{-6K}$$

$$\frac{1}{3} = e^{-6K} \Rightarrow -6K = \ln\left(\frac{1}{3}\right)$$

$$K = \frac{\ln\left(\frac{1}{3}\right)}{-6} \approx 0.1831020481$$

Sec^{-1} K

34 cont.

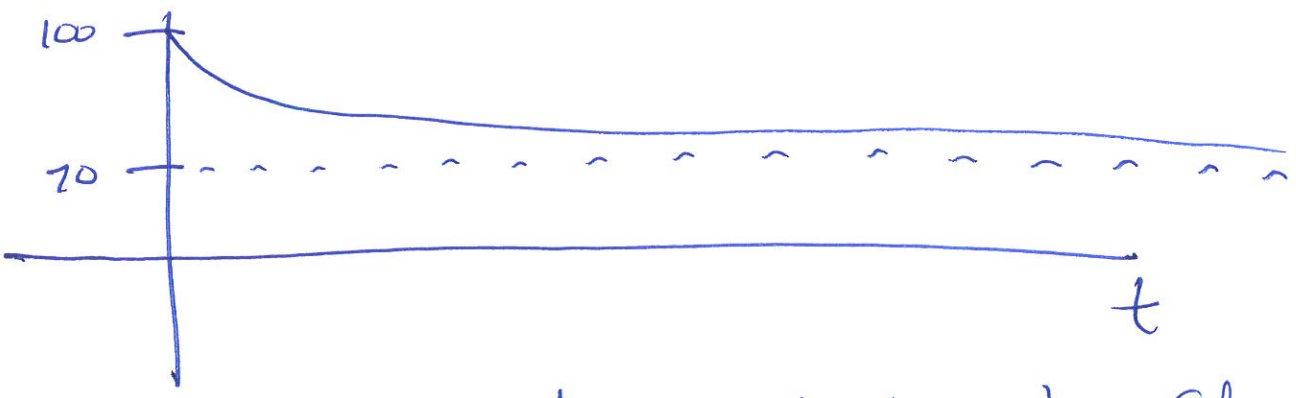
So $T(20) = 70 + 30e^{-K(20)}$

$T(20) \approx 70.770\,40142^{\circ}F$.

Note: Look at $T = 70 + 30e^{-Kt}$, $K \approx 0.183 > 0$

Then $T = 70 + \frac{30}{e^{Kt}}$

and $\lim_{t\to\infty} T = \lim_{t\to\infty}\left(70 + \frac{30}{e^{Kt}}\right) = 70$



approaches room temp. but not colder!