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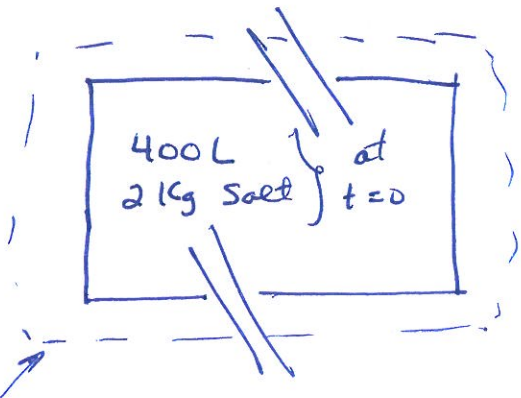
2.2

#33



Let $X = x(t) =$ Kg of Salt at time t.

$$\frac{dx}{dt} = \dot{M}_{in} - \dot{M}_{out}$$

$$\dot{M}_{in} = \left(0.3 \frac{Kg}{L}\right)\left(10 \frac{L}{min}\right) = 3 \frac{Kg}{min}$$

$$\dot{M}_{out} = \left(\frac{x}{400} \frac{Kg}{L}\right)\left(10 \frac{L}{M}\right) = \frac{x}{40} \frac{Kg}{min}$$

$$\frac{dx}{dt} = 3 - \frac{x}{40} = \frac{120 - x}{40} \qquad \text{(separable)}$$

$$\frac{40}{120 - x}\, dx = dt$$

$$40 \int \frac{1}{120 - x}\, dx = \int dt = t + C_1$$

$$\int \frac{1}{120 - x}\, dx = \frac{1}{40} t + C_2, \qquad C_2 = \frac{C_1}{40}$$

$$-\ln|120 - x| = \frac{t}{40} + C_2$$

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$$\ln|120-x| = -\frac{t}{40} + C_3, \qquad C_3 = -C_2$$

$$|120-x| = e^{-\frac{t}{40}+C_3} = e^{-\frac{t}{40}} \cdot e^{C_3}$$

$$120-x = C_4 e^{-\frac{t}{40}}$$

(Note: can drop abs. val since $120-x \geq 0$.)

$$\boxed{X = 120 - C_4 e^{-\frac{t}{40}}}$$

Now when $t=0$, $X=2$

$$2 = 120 - C_4 \Rightarrow C_4 = 118$$

$$\therefore \boxed{X = 120 - 118 e^{-t/40}}$$

Then $X(10) = 120 - 118 e^{-10/40} = 120 - 118 e^{-1/4}$

$$X(10) \approx 28.102 \text{ Kg}$$