MATH 241- EXAM 6-May 2, 2005

Instructions. Justify answers for full credit. Calculators allowed. Time given: 120 minutes.

1.[6] Use Green's theorem to compute the line integral:

$$\int_C ((1+x^2)^{3/2} - 3y)dx + (x + e^{y^3})dy,$$

where C is the oriented boundary of the region between the curves $x^2+y^2 = 2, x^2+y^2 = 5$.

2.[6] Show that the following vector field in $\mathbb{R}^3$ is conservative. Then use this fact to compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is any curve from $(0,0,0)$ to $(1,2,3)$.

$$\mathbf{F} = (e^y, xe^y + e^z, ye^z).$$

3.[6]Find the center of mass of a thin wire in the shape of a quarter-circle ($x^2 + y^2 = 4$, $x \geq 0, y \geq 0$), assuming the mass density is constant. (The *length* of the wire, of course, is π.)

4.[6] Evaluate the surface integral:

$$\int\int_S (y^2x + xz^2)dS,$$

where S is the part of the plane $x = 6+y+z$ inside the cylinder $y^2+z^2 = 9$

5.[7] Use Stokes' theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$:

$$\mathbf{F}(x,y,z) = (x+y^2, y+z^2, z+x^2).$$

C is the triangle with vertices $(1,0,0),(0,1,0),(0,0,1)$, oriented counterclockwise as seen from above.

6.[7]Use the divergence theorem to evaluate the surface integral:

$$\int\int_S (y^2 + z^2 + 2x)dS,$$

where S is the sphere $x^2+y^2+z^2 = 9$. (*Hint:* write the integrand in the form $\mathbf{F}\cdot\mathbf{N}$, for suitable $\mathbf{F}$ and $\mathbf{N}$).