MATH 241- EXAM 5-April 22, 2005

Instructions. Justify answers for full credit. Calculators allowed. SOLVE ONLY TWO OF PROBLEMS 5, 6 and 7. Time given: 60 minutes.

1.[**7**] Show that the line integral below is independent of path and compute its value (C is any path from $(-1,0)$ to $(5,1)$):

$$\int_C 2x \sin y dx + (x^2 \cos y - 3y^2)dy.$$

2.[**6**] Use Green's theorem to compute the line integral:

$$\int_C (x^3 - y^3)dx + (x^3 + y^3)dy,$$

where C is the oriented boundary of the region between the curves $x^2+y^2 = 1, x^2 + y^2 = 9$.

3.[**7**] Find the flux of the vector field $\mathbf{F}= (xy, yz, xz)$ across the surface S, described as follows: S is the part of the paraboloid $z = 4-x^2-y^2$ above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$, with the *upward* orientation.

4.[**6**] Determine whether or not the following vector field in $\mathbb{R}^3$ is conservative:

$$\mathbf{F} = (2xy, x^2 + 2yz, y^2).$$

5.[**6**] What is an 'orientable surface'? Is a surface given by an equation $F(x, y, z) = 0$ always orientable? Why?

6. [**6**] A thin wire is bent into the shape of a semicircle $x^2+y^2 = 4, y \geq 0$. If the linear mass density is constant (=1) and the total mass is 2π, find the coordinates of the center of mass of the wire.

7.[**6**] Use Green's theorem to find the *area* of the region enclosed by the hypocycloid, a simple closed curve parametrized by $\mathbf{r}(t) = (\cos^3 t, \sin^3 t), 0 \leq t \leq \pi$.