MATH 142- EXAM 3-March 16, 2005

**Instructions.** No credit for answers given without justification. Calcu-lators allowed. Time given: 60 minutes.

**1.**[4,4] For the closed curve in the (x,y) plane with parametric equations:

$$x(t) = R\cos^3 t, \quad y(t) = R\sin^3 t, \quad t \in [0, 2\pi]$$

(i) Find the total length of the curve (given: $\int_0^{\pi/2} \cos t \sin t\, dt = 1/2$)

(ii) Find the area of the region enclosed by the curve (given: $\int_0^{2\pi} \cos^2 t \sin^2 t\, dt = \pi/4$.)

**2.**[4,4,4] Consider the region $D$ bounded by the x-axis and the graph of the function:

$$y = f(x) = H(1 - x^4), \quad x \in [-1, 1].$$

(a) Find the area of $D$;

(b) Find the y-coordinate of the center of mass of $D$;

(c) Find the volume of the solid obtained by rotating D about the y-axis.

**3.**[4] Find the center of mass of the region in the first quadrant sketched below and the volume of the region obtained by rotating it about the y-axis (not necessarily in this order.) You may use *Pappus' theorem.*

**4.** [4,4] The time spent waiting in line at a certain bank is modelled by an exponential probability density function with mean 8 minutes.

(a) What is the probability that the customer is served in the first three minutes?

(b) What is the median waiting time?