March 8th, 2023

Math 351

Luís Finotti

Spring 2023

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Find the remainder of

$$a = 1977 \cdot 2000^{2023} + 2046$$

when divided by 11.

2) [20 points] Find all integers x satisfying

$$\begin{aligned} 3x &\equiv 6 \pmod{14}, \\ 5x &\equiv 3 \pmod{21}. \end{aligned}$$

3) [20 points] Prove that there are no integers x, y, such that $x^2 + y^4 = 2023$.

4) [20 points] Prove that $m \in \mathbb{Z}_{\geq 2}$ is a perfect square if and only if each of its prime factors appears an even number of times in its decomposition.

[**Note:** This was a HW problem.]

5) [20 points] Prove that if $\gcd(a, m) \nmid b$, then there is no $x \in \mathbb{Z}$ such that

$$ax \equiv b \pmod{m}.$$

[**Hint:** This was done in class. Start by converting the congruence into an *equality* of integers.]

Scratch: