Math 3A – Calculus – Final

You need to show work for all problems! Carefully write your solutions in the blue book and box your final answers. Points will be taken from messy solutions! Write the the time of your TA session in the COVER of your blue book. You only need to turn in the blue book (keep this sheet). If you are going to use a formula from your index card, say so and write the formula in your exam!
YOU NEED TO SHOW YOUR ID BEFORE RETURNING THIS EXAM!

1) (5 points each) Compute the following limits

(a) $\displaystyle\lim_{x\to\infty} \frac{(\ln(x))^3}{x^2}$

(b) $\displaystyle\lim_{x\to\infty} x^2 \cdot \ln\left(1 + \frac{3}{x^3}\right)$

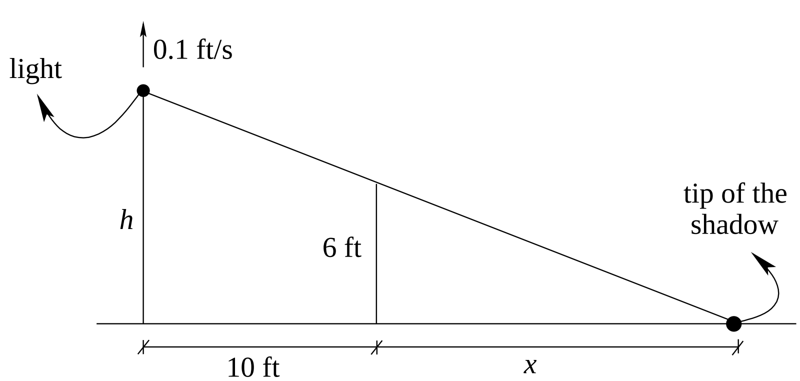
(c) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

2) (5 points each) Compute the following derivatives (no need to simplify):

(a) $\displaystyle\frac{d}{dx}\sqrt{\ln(x) + \sqrt{\cos(x)}}.$

(b) $\displaystyle\frac{d}{dx}\left([2 + \arctan(e^x)]^3\right)$

3) (10 points) *The picture below will help you understand this question.* A light bulb is being pulled up by its cord with a constant speed of 0.1 feet per second. (So, the height labeled h in the picture, is increasing by 0.1 feet per second.) A 6 foot tall man is standing 10 feet away from the light *horizontally.* (See picture.) How fast is the tip of his shadow moving toward his *feet* (not head) when the light bulb is 8 feet high? (In another words, how fast is the distance labeled x in the picture decreasing when $h = 8$?)



4) Consider the curve $y^2 = x^3 + 1$.

(a) (4 points) Compute the tangent line to the curve at the point $(2, 3)$.

(b) (3 points) Where else (besides the point $(2, 3)$) does the tangent line you found in part (a) intersect the curve?

(c) (3 points) Is the tangent line from (a) orthogonal to the curve at any of the other points of intersection you found in part (b)?

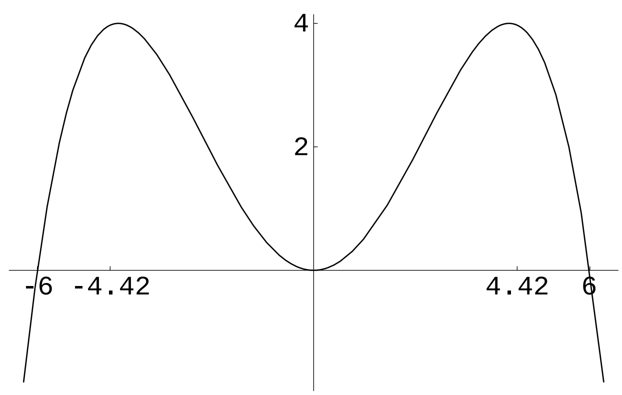
5) (10 points) Find the *absolute* maximum and minimum of $f(x) = \dfrac{x^2+1}{x^2-5}$ in the interval $[-2, 1]$.

6) (10 points) Prove the well-known formula

$$\sin^2(x) + \cos^2(x) = 1.$$

7) Suppose that the graph of $f'(x)$ (i.e., the graph of the *derivative* of $f(x)$, *not the graph of* $f(x)$ *itself*) is given below.

Graph of $f'(x)$



The questions below refer to $f(x)$ itself, *not* the derivative!

(a) (3 points) On what interval is $f(x)$ increasing? Where is it decreasing?

(b) (2 points) What values of x give the local maxima for $f(x)$ (if any)? What about local minima?

(c) (3 points) On what interval is $f(x)$ concave up? Where is it concave down?

(d) (2 points) Does $f(x)$ have inflection points? If so, for what values of x?

8) Let $f(x) = \dfrac{1 - x^2}{x^3}$.

(a) (2 points) What is the domain of the function? Give the equations for the vertical asymptotes, if any.

(b) (2 points) Does $f(x)$ have a horizontal asymptote or oblique (also called slant) asymptote? If so, give its equation.

(c) (2 points) Find the x and y-intercepts, if any.

(d) (5 points) Where is the function increasing and where is it decreasing? (Use intervals to give your answer.)

(e) (2 points) What are the local maxima and local minima for the function? (You have to give the x and y coordinates of those points.)

(f) (5 points) Where is $f(x)$ concave up? Where is it concave down? (Again, use intervals to give your answer.)

(g) (2 points) Give the coordinates of the points of inflection, if any.

(h) (5 points) Sketch the graph of the function.

9) (10 points) You want to make a box (with no lid) out of a 3' by 3' cardboard by cutting squares from the edges and then folding it. (See picture below.) What is the size of the side of the square to be cut off (represented by x in the picture) so that the volume of the resulting box is maximal? What is this maximal volume? *Don't forget you have two questions to answer!!!*