# List of Topics for the CAM Preliminary Exam

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## Numerical linear algebra

### Linear algebra review

1. The field of complex numbers
2. Vector spaces
3. Normed spaces
4. Inner product spaces
5. Gram–Schmidt orthogonalization
6. Linear operators and matrices
7. Matrix norms
8. Eigenvalues and spectral decomposition

### The singular value decomposition

1. Reduced and full singular value decompositions
2. Existence and uniqueness of the SVD
3. Further properties of the SVD
4. Low rank approximations

### Systems of linear equations

1. Solution of simple systems
2. LU factorization
3. Gaussian elimination with column pivoting
4. Implementation
5. Special matrices

## Norms and matrix conditioning

1. The spectral radius
2. Conditioning

## Linear least squares problem

1. Linear least squares: Full rank setting
2. Projection matrices
3. Linear least squares: The rank–deficient case
4. QR Factorization
5. The Moore–Penrose pseudo–inverse
6. The modified Gram–Schmidt process
7. Householder reflectors

## Linear iterative methods

1. Linear iterative schemes
2. Spectral convergence theory
3. Matrix splitting methods
4. Richardson's method
5. Relaxation methods
6. The Householder–John criterion
7. Convergence in energy norm

## Variational and Krylov subspace methods

1. Basic facts about HPD matrices
2. Gradient descent methods
3. The steepest descent method
4. The conjugate gradient method

### Eigenvalue problems

1. Estimating eigenvalues using Gershgorin discs
2. Stability
3. The Rayleigh quotient for Hermitian matrices
4. Power iteration methods
5. Reduction to Hessenberg form
6. The QR method

## Nonlinear equations and optimization

### Solution of nonlinear equations

1. Bisection method
2. Fixed points and contraction mappings
3. Newton's method in one space dimension
4. Newton's method in several dimensions

## Initial value problems for ordinary differential equations

### Single–step methods

1. Single–step approximation schemes
2. Consistency and convergence of some single–step approximation

### Runge–Kutta methods

1. Simple two–stage schemes
2. Definition and basic properties
3. Collocation methods

### Linear multi–step methods

1. Consistency
2. Adams–Bashforth and Adams–Moulton methods
3. Backward differentiation formula methods
4. Zero stability
5. Convergence of linear multistep methods
6. Dahlquist theorems

### Stiff systems of ordinary differential equations and linear stability

1. The linear stability domain and A–stablity
2. A–Stability of Runge–Kutta schemes
3. A–stability of linear multi–step methods

## Boundary and initial boundary value problems

### Finite difference methods for elliptic problems

1. Grid functions and finite difference operators
2. Consistency and stability of finite difference schemes
3. The Poisson problem in one dimension
4. Elliptic problems in one dimension
5. The Poisson problem in two dimensions

### Finite element methods for elliptic problems

1. The Galerkin method
2. The finite element method in one dimension
3. The finite element method in two dimensions

### Approximation of the diffusion equation

1. Diffusion in 1D
2. $L^\infty_\tau(L^\infty_h)$ stability and convergence
3. $L^\infty_\tau(L^2_h)$ stability and convergence
4. An advection-diffusion equation
5. An energy method
6. von Neumann stability analysis

### The advection and wave equations

1. The linear advection equation
2. Positivity and max-norm dissipativity
3. Advection on a periodic spatial domain
4. The wave equation and hyperbolic systems

## References

### Main


- A.J. Salgado, S.M. Wise. *Graduate Numerical Analysis: A Modern Introduction.* Set of notes available at `http://www.math.utk.edu/~swise/Site/Numerical_Analysis.html`.


### Supplementary


1. K. Atkinson and W. Han. Theoretical Numerical Analysis. Texts in Applied Mathematics 39. Springer-Verlag, New York, NY, USA, third edition, 2009.
2. D. Braess. Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics. Cambridge University Press, Cambridge, UK, third edition, 2007.
3. S.C. Brenner and L.R. Scott. The Mathematical Theory of Finite Element Methods. Texts in Applied Mathematics 15. Springer-Verlag, Berlin, Germany, third edition, 2007.
4. J.C. Butcher. Numerical Methods for Ordinary Differential Equations. John Wiley and Sons, Chichester, UK, second edition, 2008.
5. P.G. Ciarlet. Introduction to Numerical Linear Algebra and Optimisation. Cambridge University Press, Cambridge, UK, 1989.

6. P.G. Ciarlet. The finite element method for elliptic problems, volume 40 of Classics in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2002.

7. J.W. Demmel. Applied numerical linear algebra. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1997.

8. J.E. Dennis, Jr. and R.B. Schnabel. Numerical methods for unconstrained optimization and nonlinear equations, volume 16 of Classics in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1996. Corrected reprint of the 1983 original.

9. A. Ern and J.-L. Guermond. Theory and practice of finite elements, volume 159 of Applied Mathematical Sciences. Springer-Verlag, New York, 2004.

10. W. Gautschi. Numerical Analysis. Birkhauser-Verlag, New York, NY, USA, second edition, 2012.

11. G.H. Golub and C.F. Van Loan. Matrix Computations. Johns Hopkins University Press, Baltimore, MD, USA, fourth edition, 2013.

12. L.A. Hageman and D.M. Young. Applied Iterative Methods. Academic Press, San Diego, CA, USA, 1981.

13. E. Hairer, S. Norsett, and G. Wanner. Solving Ordinary Differential Equations I: Nonstiff Problems. Springer-Verlag, Berlin, Germany, second edition, 1993.

14. E. Isaacson and H.B. Keller. Analysis of Numerical Methods. John Wiley and Sons, New York, NY, USA, 1966.

15. A. Iserles. A First Course in the Numerical Analysis of Differential Equations. Cambridge University Press, Cambridge, UK, second edition, 2009.

16. B.S. Jovanoviv́ and E. Süli. Analysis of finite difference schemes, volume 46 of Springer Series in Computational Mathematics. Springer, London, 2014. For linear partial differential equations with generalized solutions.

17. C.T. Kelley. Iterative methods for linear and nonlinear equations, volume 16 of Frontiers in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1995. With separately available software.

18. R. Kress. Numerical Analysis. Springer-Verlag, Berlin, Germany, 1998.

19. R.J. LeVeque. Finite Volume Methods for Hyperbolic Problems. Cambridge University Press, Cambridge, UK, 2002.

20. J.M. Ortega and W.C. Rheinboldt. Iterative solution of nonlinear equations in several variables, volume 30 of Classics in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2000. Reprint of the 1970 original.

21. B.N. Parlett. The Symmetric Eigenvalue Problem. Prentice-Hall, Englewood Cliffs, NJ, USA, 1980.

22. Y. Saad. Iterative methods for sparse linear systems. Society for Industrial and Applied Mathematics, Philadelphia, PA, second edition, 2003.

23. A.A. Samarskii. The theory of difference schemes, volume 240 of Monographs and Textbooks in Pure and Applied Mathematics. Marcel Dekker, Inc., New York, 2001.

24. L.R. Scott. Numerical Analysis. Princeton University Press, Princeton, NJ, USA, 2011.

25. J. Stoer and R. Bulirsch. Introduction to Numerical Analysis. Springer-Verlag, Berlin, Germany, third edition, 2002.

26. E. Süli and D.F. Mayers. An Introduction to Numerical Analysis. Cambridge University Press, Cambridge, UK, 2003.

27. L.N. Trefethen and D. Bau. Numerical Linear Algebra. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997.

28. D.M. Young. Iterative Solution of Large Linear Systems. Academic Press, Cambridge, MA, USA, 1971.