ALGEBRA

The preliminary examination in algebra covers topics in group theory; ring theory; module theory; and field theory and Galois theory. Most, and usually all, the required topics are discussed annually in Mathematics 551-552. Studying appropriate portions of the texts by Hungerford and Lang would certainly be adequate preparation for the examination; less comprehensive (but possibly more readable) texts are those by Herstein ("Topics in Algebra"), Jacobson ("Algebra I"), Paley and Weichsel, and van der Waerden (volume I). Students wishing specialized sources are referred in group theory to appropriate sections of the books by Baumslag and Chandler (Schaum's), Rotman, and Scott; in rings and modules, to Lambek and McCoy ("The Theory of Rings"); for background in linear algebra, to Halmos ("Finite Dimensional Vector Spaces"), Hoffman and Kunze, and Jacobson ("Lectures in Abstract Algebra," volume II); in field theory and Galois theory to E. Artin ("Galois Theory") and Kaplansky (part I in "Rings and Fields"). Such a list is necessarily incomplete, for there is no lack of good texts on abstract algebra and its areas of specialization.

In addition to the specific items listed below, students should know the basic concepts applicable to several areas (such as homomorphism, kernel, isomorphism theorems, generating sets, direct product, universal mapping property, center, etc.). A nodding acquaintance with the rudiments of categorical language is desirable; for this, students are advised to browse through the early portions of the books by MacLane ("Categories...") and Mitchell, or to read the appropriate chapter in Hungerford.

Group Theory: basic examples (dihedral and quaternion groups, symmetric groups, alternating groups, group of invertible elements of ring, matrix groups, automorphism groups, etc.), cyclic groups, finitely generated abelian groups, free abelian groups, types of subgroup (normal, commutator, normalizer, Sylow, etc.), Cayley's theorem and generalizations, simple group, simplicity of A_5, composition series, Jordan-Hölder theorem, finite solvable group, class equation, p-group, Sylow Theorems, classification of groups of low order.

Ring Theory: endomorphism ring of an abelian group, prime and maximal ideals, Chinese Remainder theorems, commutativity, radicals, zero-divisor, integral domain, division ring, field, local ring, polynomial rings, division algorithm, Euclidean domain, principal ideal domain, unique factorization domain, noetherian ring, localization, quotient fields, primitive polynomials and the lemma of Gauss, Eisenstein's criterion.

Module Theory: exact sequences, the Hom functors, basis, free module, projective modules, tensor product of modules and of homomorphisms, tensor algebra, vector space, dual space, chain conditions on modules, structure of finitely generated modules over a principal ideal domain (including the fundamental theorem of abelian groups).

Field Theory and Galois Theory: prime fields, characteristic, algebraic extensions, separable extensions, purely inseparable extensions, splitting fields, algebraic closures, constructible numbers, purely transcendental extensions, transcendence degree, Galois extensions, fundamental theorem of Galois theory, computation of Galois groups, symmetric polynomials, structure of finite fields, perfect fields, primitive element theorem, fundamental theorem of algebra, cyclotomic extensions, solvability by radicals.

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