All vector spaces are assumed to be finite dimensional.

1. Suppose that \( \{v_1, \ldots, v_m\} \) is a linearly independent subset of the vector space \( V \), and that \( w \in V \). Show that the span of \( \{v_1 + w, \ldots, v_m + w\} \) has dimension at least \( m - 1 \).

2. Let \( A \) be a \( 4 \times 4 \) matrix with entries in the complex numbers, satisfying \( A \neq 0, A^2 = 0 \). Determine the possible Jordan canonical forms for \( A \).

3. Suppose that \( U, W \) are both 4-dimensional subspaces of \( \mathbb{C}^6 \). Prove that there exist two vectors in \( U \cap W \) such that neither of these vectors is a scalar multiple of the other.

4. Let \( T : V \to V \) be an invertible linear transformation. Prove that a vector \( v \in V \) is an eigenvector of \( T \) if and only if it is an eigenvector of \( T^{-1} \).

5. Let \( T : \mathbb{C}^3 \to \mathbb{C}^3 \) be a linear transformation with eigenvalues 2, 3, 5. Show that there exists a linear transformation \( S : \mathbb{C}^3 \to \mathbb{C}^3 \) with \( S^2 = T \).

6. A linear map \( T : \mathbb{C}^3 \to \mathbb{C}^3 \) is defined by \( T(z_1, z_2, z_3) = (2z_2, 0, 3z_1) \). Prove that \( T \) does not have a square root, i.e. show that there does not exist a linear map \( S : \mathbb{C}^3 \to \mathbb{C}^3 \) such that \( S^2 = T \).