PAlmetto Number Theory Series XXVIII

University of Tennessee, September 16-17, 2017

Titles and Abstracts

Nick Andersen UCLA

Title: Polyharmonic Maass forms

Abstract: Polyharmonic Maass forms are nonholomorphic modular forms which are annihilated by a (possibly fractional) power of the hyperbolic Laplacian. My talk will be an example-driven introduction to polyharmonic Maass forms, focusing on a function constructed by Duke, Imamoglu, and Toth, and one constructed by myself, S. Ahlgren, and D. Samart. The coefficients of the former encode class numbers of real and imaginary quadratic forms, while the coefficients of the latter are related to partition statistics. Time permitting, I will state some more general results on shifted polyharmonic Maass forms which are joint with J. Lagarias and R. Rhoades.

Victor Manuel Aricheta Emory University

Title: Supersingular Elliptic Curves and Sporadic Groups

Abstract: We give a procedure for computing generalized supersingular polynomials—polynomials whose roots correspond to supersingular elliptic curves with additional structure—and determine when these polynomials split completely. This analysis yields geometric characterizations of the primes dividing the orders of certain sporadic simple groups (e.g. baby monster, Fischer, Conway, etc.); this is an extension of Ogg's geometric characterization of the primes dividing the order of the monster. Finally, we also present evidence for a role of supersingular elliptic curves in umbral moonshine.

Olivia Beckwith Emory University

Title: Indivisibility of class numbers of imaginary quadratic fields

Abstract: I quantify a recent theorem of Wiles by proving an estimate for the number of negative fundamental discriminants down to -X whose class numbers are indivisible by a given prime and whose imaginary quadratic fields satisfy almost any given finite set of local conditions. This estimate matches the best results in the direction of the Cohen-Lenstra heuristics for the number of imaginary quadratic fields with class number indivisible by a given prime. I will also show how this result can be applied to study rank 0 twists of certain elliptic curves.

Alex Berkovich University of Florida

Title: Euler's Pentagonal theorem and $(q)_m$

Abstract: I will show how to use Euler's Pentagonal theorem to prove that the q-series coefficients of $(q)_m$ are in $-1, 0, 1$ iff $m = 0, 1, 2, 3, 5$. This talk is based on my recent joint work with Ali Uncu.

Stevo Bozinovski South Carolina State University

Title: Divergent series: an approach

Abstract: The talk will presents an approach toward study of divergent series. It parallels the approach based on summation methods and produces some different results. The following result will be discussed: Dirichlet eta(-2) = Riemann zeta(-1) in case both series have same odd number of terms. Dirichlet eta(-2) = -Riemann zeta(-1) in case both series have same even number of terms. This is a joint work with Adrijan Bozinovski.

Madeline Locus Dawsey Emory University

Title: A New Formula for Chebotarev Densities

Abstract: We give a new formula for the Chebotarev densities of Frobenius elements in Galois groups. This formula is given in terms of smallest prime factors $p_{\min}(n)$ of integers $n \geq 2$. More precisely, let C be a conjugacy class of the Galois group of some finite Galois extension K of $\mathbb{Q}$. Then we prove that

$$-\lim_{X\to\infty} \sum_{\substack{2\leq n\leq X \\ \left[\frac{K/\mathbb{Q}}{p_{\min}(n)}\right]=C}} \frac{\mu(n)}{n} = \frac{\#C}{\#G}.$$

This theorem is a generalization of a result of Alladi which asserts that largest prime divisors $p_{\max}(n)$ are equidistributed in arithmetic progressions modulo an integer k, which occurs when K is a cyclotomic field $\mathbb{Q}(\zeta_k)$.

Dong Dong University of Illinois at Urbana-Champaign

Title: Polynomial Roth type theorems in Finite Fields

Abstract: Recently, Bourgain and Chang established a nonlinear Roth theorem in finite fields: any set (in a finite field) with not-too-small density contains many nontrivial triplets of the form x, $x+y$, $x+y^2$. The key step in Bourgain-Chang's proof is a 1/10-decay estimate of some bilinear form. We slightly improve the estimate to a 1/8-decay (and thus a better lower bound for the density is obtained). Our method is also valid for 3-term polynomial progressions x, $x+P(y)$, $x+Q(y)$. Besides discrete Fourier analysis, algebraic geometry (theorems of Deligne and Katz) is used. This is a joint work with Xiaochun Li and Will Sawin.

Frank Garvan University of Florida, Gainesville

Title: New Ramanujan Mock Theta Function Identities

Abstract: In his last letter to Hardy, Ramanujan defined ten mock theta functions of order 5 and three of order 7. He stated that the three mock theta functions of order 7 are not related. We show how we discovered new identities for the fifth order functions and there are actually surprising relationships between the order 7 functions.

Zhenchao Ge The University of Mississippi

Title: The least rational prime that splits completely in a Galois number field

Abstract: The study of the distribution of quadratic residues and nonresidues is a classical problem that can be traced back to Gauss. We will discuss a generalization of this problem and prove the first nontrivial bound for the least rational prime that splits completely in certain nonabelian Galois number fields. This complements earlier work of Linnik and Vinogradov and of Pollack who studied this problem in the quadratic and abelian number field settings, respectively. This is joint work with M. Milinovich and P. Pollack.

Jon Grantham IDA/CCS

Title: Parallel Computation of Primes of the Form $x^2 + 1$

Abstract: It has long been an open question whether or not there are infinitely many primes of the form x^2+1. Marek Wolf and Robert Gerbeicz recently computed all primes of the form $x^2 + 1$ up to 10^{26}. We extend the list up to $6.25 \cdot 10^{28}$ through a parallel implementation.

Catherine Hsu University of Oregon

Title: Higher Eisenstein Congruences

Abstract: Let $p \geq 3$ be prime. For squarefree level $N > 6$ and weight $k = 2$, we use a commutative algebra result of Berger, Klosin, and Kramer to bound the depth of Eisenstein congruences modulo p (from below) by the p-adic valuation of the numerator of $\frac{\varphi(N)}{24}$. We then show that if N has at least three prime factors and some prime $p \geq 5$ divides $\varphi(N)$, the Eisenstein ideal is not locally principal. Time-permitting, we will illustrate these results with explicit computations as well as discuss generalizations to higher weights.

Christopher Keyes Tufts University

Title: Growth of points on hyperelliptic curves

Abstract: Let C be a hyperelliptic curve over $\mathbb{Q}$ of genus g. Granville conjectured that such a curve gains a point over about $X^{1/(g+1)}$ quadratic fields of discriminant up to X. Motivated by this conjecture, we aim to count the number of degree n number fields of discriminant up to X over which C gains a point. For C of odd degree and sufficiently large n we give an unconditional lower bound for the number of such fields. Surprisingly, the exponent of X in this count does not decay to zero as n and g grow large. This builds on work of Ellenberg and Venkatesh on counting number fields, and recent work of Lemke Oliver and Thorne on the analogous problem for elliptic curves.

Thai Hoang Le University of Mississippi

Title: Additive bases in groups

Abstract: Let $\mathbb{N}$ be the set of all nonnegative integers. A set $A \subset \mathbb{N}$ is called a basis of $\mathbb{N}$ if every sufficiently large integer is a sum of h elements from A, for some h. The smallest such h is called the order of A. For example, the squares form a basis of order 4 and the primes conjecturally form a basis of order 3 of $\mathbb{N}$. Erdős and Graham asked the following questions. If A is a basis of $\mathbb{N}$ and $a \in A$, when is $A \setminus \{a\}$ still a basis? It turns out that this is the case for all $a \in A$ with a finite number of exceptions. If $A \setminus \{a\}$ is still a basis, what can we say about its order? These questions and related questions have been extensively studied. In this talk, we address these questions in the more general setting of an abelian group in place of $\mathbb{N}$. This is joint work with Victor Lambert and Alain Plagne.

Spencer Leslie Boston College

Title: A Generalized Lifting Via Higher Theta Functions

Abstract : The Jacobi theta function θ transforms with respect to a multiplier system that includes a quadratic Kronecker symbol $\left(\frac{c}{d}\right)$. This tells us that it may thought of as a modular form on the double cover of SL(2). Generalizations of θ to double covers of symplectic groups play a fundamental role: they have very few non-zero Fourier coefficients, which enables them to serve effectively as integral kernels to lift automorphic forms from one group to another. This is the theory of the theta correspondence, which has had huge impact on number theory and representation theory.

For higher-degree covers of symplectic groups, we have generalized theta representations and it is natural to ask if these "higher" theta functions play a similar role in the theory of metaplectic forms. In this talk, I will discuss new lifting of automorphic representations on the 4-fold cover of symplectic groups using such theta functions. A key feature is that this lift produces counterexamples of the generalized Ramanujan conjecture, which motivates a connection to the emerging "Langlands program for covering groups" by way of Arthur parameters. The crucial fact allowing this lift to work is that theta functions for the 4-fold cover still have few non-vanishing Fourier coefficients, which fails for higher-degree covers.

Huixi Li Clemson University

Title: Frobenius Distributions in Short Intervals for CM Elliptic Curves

Abstract: For an elliptic curve $E/\mathbb{Q}$, Hasse's theorem asserts that $\#E(\mathbb{F}_p) = p + 1 - a_p$, where $|a_p| \leq 2\sqrt{p}$. Assuming that E has complex multiplication, a theorem by Hecke and Deuring gives the asymptotic formula for primes p for which $2\alpha\sqrt{p} \leq a_p \leq 2\beta\sqrt{p}$, $a_p \neq 0$ and $-1 \leq \alpha \leq \beta \leq 1$. Recently we study the asymptotics for primes p where a_p is in subintervals of the Hasse interval of measure $o(\sqrt{p})$. In particular, given a function $f = o(1)$ satisfying some mild conditions, we provide counting functions for primes where $a_p \in [2\sqrt{p}(c - f(p)), 2c\sqrt{p}]$, where $c \in (0, 1]$ is a constant. The results generalize the work by James and Pollack on champion primes. This is joint work with Agwu, Harris, James and Kannan.

Micah Milinovich University of Mississippi

Title: Fourier optimization and gaps between consecutive primes

Abstract: I will discuss the proofs of the strongest known asymptotic and explicit estimates in the classical problem of bounding the maximum gap between consecutive primes assuming the Riemann hypothesis. These proofs involve three main ingredients: the explicit formula relating the primes to the zeros of the Riemann zeta-function, the size of the constant in the Brun-Titchmarsh inequality, and Fourier optimization. This is joint work with E. Carneiro (IMPA) and K. Soundararajan (Stanford).

Michael Mossinghoff Davidson College

Title: The Lind-Lehmer constant for certain p-groups

Abstract: In 2005, Lind formulated an analogue of Lehmer's well-known problem regarding the Mahler measure for general compact abelian groups, and defined a Lehmer constant for each group. Since then, this Lind-Lehmer constant has been determined for many finite abelian groups, including all but a thin set of finite cyclic groups. We establish some new congruences satisfied by the Lind Mahler measure for finite p-groups, and use them to determine the Lind-Lehmer constant for many groups of this form. We also develop an algorithm that determines a small set of possible values for a given p-group of a particular form. This method is remarkably effective, producing just one permissible value in all but a handful of trials. This is joint work with Dilum De Silva, Vincent Pigno, and Chris Pinner.

Fred Moxley Louisiana State University

Title: Eighth Hilbert Problem

Abstract: The Hamiltonian of a quantum mechanical system has an affiliated spectrum. If this spectrum is the sequence of prime numbers, a connection between quantum mechanics and the nontrivial zeros of the Riemann zeta function can be made. In this case, the Riemann zeta function is analogous to chaotic quantum systems, as the harmonic oscillator is for integrable quantum systems. Such quantum Riemann zeta function analogies have led to the Bender-Brody-Müller (BBM) conjecture, which involves a non-Hermitian Hamiltonian that maps to the zeros of the Riemann zeta function. If the BBM Hamiltonian can be shown to be Hermitian, then the Riemann Hypothesis follows. As such, herein we perform a symmetrization procedure of the BBM Hamiltonian to obtain a unique Hermitian Hamiltonian that maps to the zeros of the Riemann zeta function, and discuss the eigenvalues of the results. Moreover, a second quantization of the resulting Schrödinger equation is performed, and a convergent solution for the nontrivial zeros of the Riemann zeta function is obtained. Finally, it is shown that the real part of every nontrivial zero of the Riemann zeta function converges at $\sigma = 1/2$.

Bach Nguyen Louisiana State University

Title: Discriminant of Quantum Schubert Cell Algebras via Poisson Geometry

Abstract: The notion of discriminant is an important tool in number theory, algebraic geometry and noncommutative algebra. However, in concrete situations, it is difficult to compute and this has been done for few noncommutative algebras by direct methods. In this talk, we will describe a general method for computing noncommutative discriminants which relates them to representation theory and Poisson geometry. As an application we will provide explicit formulas for the discriminants of the quantum Schubert cell algebras at roots of unity. This is a joint work with Kurt Trampel and Milen Yakimov.

Spencer Saunders University of South Carolina

Title: Polynomials of Small Mahler Measure with No Newman Multiples

Abstract: A Newman polynomial is a polynomial with coefficients in $0, 1$ and with constant term 1. It is known that the roots of a Newman polynomial must lie in the slit annulus $\{z \in \mathbb{C} : \phi^{-1} < |z| < \phi\} \setminus \mathbb{R}+$ where ϕ denotes the golden ratio; however, it is not guaranteed that all polynomials whose roots lie in this slit annulus divide a Newman polynomial. The Mahler measure of a monic polynomial is defined to be the product of the absolute values of those roots of the polynomial which are greater than 1. K. Hare and M. Mossinghoff have asked whether there is a $\sigma > 1$ such that if a polynomial $f(z) \in \mathbb{Z}[z]$ has Mahler measure less than σ and has no nonnegative real roots, then it must divide a Newman polynomial. In this thesis, we present a new upper bound on such a σ if it exists. We also show that there are infinitely many monic polynomials that have distinct Mahler measures which all lie below ϕ, have no nonnegative real roots, and have no Newman multiples. Finally, we consider a more general notion in which multiples of polynomials are considered in $\mathbb{R}[z]$ instead of $\mathbb{Z}[z]$.

Armin Straub University of South Alabama

Title: A modular supercongruence for ${}_6F_5$: An Apéry-like story

Abstract: We discuss congruences between truncated hypergeometric series and modular forms. Specifically, we discuss a supercongruence modulo p^3 between the pth Fourier coefficient of a weight 6 modular form and a truncated ${}_6F_5$-hypergeometric series. The story is intimately tied with Apéry's proof of the irrationality of $\zeta(3)$. This is joint work with Robert Osburn and Wadim Zudilin.

Frank Thorne University of South Carolina

Title: Uniformity in Landau's method

Abstract: If $L(s) = \sum_n a(n)n^{-s}$ is a "nice zeta function", then a classical method of Landau allows one to obtain estimates for the partial sums $\sum_{n<X} a(n)$, with power saving error terms. I will give an overview of the method and some of its classical applications.

I will then give an overview of joint work with David Lowry-Duda and Takashi Taniguchi, where we obtain such a result with substantial uniformity in the error term. The result is quite technical, so mostly I will just explain why we think it's interesting.

John Voight Dartmouth College

Title: Heuristics for units in number rings

Abstract: Units are precious elements in number rings–buried within, entwined with the class group. Refined questions about the structure of units remain difficult to answer, for example: how often does it happen that all totally positive units are squares?

In this talk, we present heuristics for signatures of unit groups inspired by the Cohen-Lenstra heuristics for class groups, but involving an lustrous structure of number rings we call the 2-Selmer signature map. This is joint work with David S. Dummit.

Ian Wagner Emory University

Title: Harmonic Maass form eigencurves

Abstract: We construct two families of harmonic Maass Hecke eigenforms. Using these families, we construct p-adic harmonic Maass forms in the sense of Serre. These forms answer a question of Mazur about the existence of an "eigencurve-type" object in the world of harmonic Maass forms.

Tom Wright Wofford College

Title: An almost-theorem for Carmichael numbers with limited numbers of prime factors

Abstract: In 2015, we were able to prove that, if one assumed weakened version of Dickson's conjecture, one could prove that there exists a bound B such that there are infinitely many Carmichael numbers with at most B prime factors. In this talk, we weaken the required conjecture further, *almost* reaching the point where the theorem is unconditional.

Michael Zieve University of Michigan

Title: Near-injectivity of rational functions over number fields

Abstract: I will show that every univariate polynomial with rational coefficients induces a function on the rational numbers which is at most 6-to-1 over all but finitely many values. I will then present progress towards a rational function analogue of this result, which would comprise a vast generalization of Mazur's theorem on uniform boundedness of rational torsion on elliptic curves. This progress relies on a new description of all complex rational functions whose global behavior is "non-random" in a certain precise sense, which in turn has applications to topics such as refinements of Hilbert's irreducibility theorem and image sizes of rational functions over finite fields.