Since \( n(t) \leq 0 \ \forall t \), car is never moving backwards

Where velocity is constant the position function is linear

\[ y = x^2 \Rightarrow \frac{dy}{dx} = 2x \leftarrow \text{slope of tangent line} \]

\[ m_1 = 2x_1, \ \text{and} \ m_2 = 2x_2 \leftarrow \text{slopes of tangent lines 1 and 2} \]

Given: \( m_1 = \frac{-1}{m_2} \leftarrow \text{perpendicular} \Rightarrow \text{negative reciprocal} \)

So: \( 2x_1 = \frac{-1}{2x_2} \rightarrow 4x_1x_2 = -1 \rightarrow x_1, x_2 = \frac{-1}{4} \text{ or } x_2 = 4x_1 \)

Line 1: \( y - x_1^2 = 2x_1(x - x_1) \)

Line 2: \( y - \frac{1}{16x_1^2} = \frac{-1}{2x_2} (x + \frac{1}{4x_1}) \)

Intersect when \( x = 0 \), so plug in \( x = 0 \) and set equal:

\[ y = -2x_1^2 + x_1^2 = -x_1^2 \]

\[ y = \frac{1}{16x_1^2} - \frac{1}{8x_1^2} = -\frac{1}{16x_1^2} \]

\[ x_1^2 = \frac{1}{16x_1^2} \Rightarrow x_1 = \pm \frac{1}{2} \]

\[ 4x_1^2 = 1 \]

More!
So far I have found \( x_1 = \frac{-1}{2} \)

When \( x_1 = \frac{1}{2} \), point: \( \left( \frac{1}{2}, \frac{1}{4} \right) \) and slope \( 2(\frac{1}{2}) = 1 \)

So line 1: \( y - \frac{1}{4} = 1(x - \frac{1}{2}) \) or \( y = x - \frac{1}{4} \)

Since \( x_1x_2 = -\frac{1}{4} \) (from previous page)

\[
\frac{1}{2} \cdot x_2 = -\frac{1}{4}
\]

so \( x_2 = -\frac{1}{2} \)

line 2 contains point \( (-\frac{1}{2}, \frac{1}{4}) \) and slope \( 2(-\frac{1}{2}) = -1 \)

line 2: \( y - \frac{1}{4} = -1(x + \frac{1}{2}) \) or \( y = -x - \frac{1}{4} \)

Both these lines are tangent lines to \( y = x^2 \), they are \( \perp \) lines, and they intersect on the \( y \)-axis at \( (0, -\frac{1}{4}) \).
a) Apply the product rule to find \((fgh)'\):

\[
(fgh)' = (fg \cdot h)' = fg \cdot h' + h \cdot (fg)'
\]

\[
= fg \cdot h' + h \cdot [f \cdot g' + g \cdot f'] = fgh' + fg'h + f'gh.
\]

b) When \(f = g = h\), then \((fgh)' = (f)^3\)

So \(\frac{d}{dx}[f(x)]^3 = fff' + ff'f + f'ff\) (by a)

\[
= 3fff' = 3(f(x))^2 \cdot f'(x)
\]

c) \(f(x) = e^{3x} = (e^x)^3\) by laws of exponents.

\[
f'(x) = 3(e^x)^2 \cdot e^x\) by part b

Simplifying: \(3e^{2x}e^x = 3e^{x+2} = 3e^{3x}\).

This agrees with what we saw Monday with the chain rule.
a) Did in class

b) \( y = \frac{1}{s + ke^s} \)
\[
\frac{dy}{dx} = -\frac{1 + ke^s}{(s + ke^s)^2}
\]

c) Let \( y = x^{-n} \) where \( n \in \mathbb{Z}^+ \).

Then \( y = x^{-n} \). By the reciprocal rule (part a), we know that \( \frac{dy}{dx} = -\frac{n}{(x^n)^2} \).

Simplifying: \( \frac{-nx^{n-1}}{(x^n)^2} = -\frac{nx^{n-1}}{x^{2n}} = -nx^{-1} \cdot x^{-2n} \)

\( = -n \cdot x^{-1 + (-2n)} = -n \cdot x^{-n-1} \). \( \square \)

(the simplifying was all by rules of exponents.)
\[ F = \frac{GmM}{r^2} = \text{constant in this problem} \]

\[ a) \quad \frac{dF}{dr} = GmM \cdot (-2) \frac{1}{r^3} = -2GmM \frac{1}{r^3} \]

This gives the change in force in terms of the change in distance. It is negative because the force gets smaller (decreases) as you get further away.

\[ b) \quad \text{When } r = 20,000, \quad \frac{dF}{dr} = -2. \quad \text{Find } \frac{dF}{dr} \text{ when } r = 10,000. \]

\[-2 = -2GmM \frac{1}{(20,000)^3} \quad \Rightarrow \quad GmM = (20,000)^3\]

\[ \text{So } \frac{dF}{dr} \bigg|_{r=10,000} = -2GmM \frac{1}{(10,000)^3} = -2 \frac{(20,000)^3}{(10,000)^3} = -16 \]

When \( r = 10,000 \text{ km} \), the force decreases at a rate of \(-16 \text{ N/km}\).