p139 #42:

a) [can see on calc. screen.]

b) \[ \lim_{x \to \infty} \frac{P(x)}{Q(x)} = \lim_{x \to \infty} \frac{3x^5 - 5x^3 + 2x}{3x^5} \]

\[ = \lim_{x \to \infty} 1 - \frac{5}{3x^2} + \frac{2}{3x^4} = 1 \]

p139 #48:

a) \[ \lim_{t \to \infty} v(t) = \lim_{t \to \infty} v^* - \frac{v^*}{e^{gt/v^*}} \]

As \( t \to \infty \), \( e^{gt/v^*} \to \infty \), so \( \frac{v^*}{e^{gt/v^*}} \to 0 \)

so \( \lim_{t \to \infty} v(t) = v^* \).

b) \( v(t) = 1 - e^{-9.8t} \)

\( 0.99 = 1 - e^{-9.8t} \)

\[ e^{-9.8t} = 0.01 \]

\[ -9.8t = \ln(0.01) \]

\[ t = \frac{\ln(0.01)}{-9.8} \approx 0.47 \text{ s} \]
p139 #50

a) \[ \lim_{x \to \infty} \frac{4x^2 - 5x}{2x^2 + 1} = \lim_{x \to \infty} \frac{4 - \frac{5}{x}}{2 + \frac{1}{x^2}} = \frac{4}{2} = 2. \]

b) \[ \frac{4x^2 - 5x}{2x^2 + 1} > 1.9 \quad \text{when} \quad x > 25.4 \]

\[ \frac{4x^2 - 5x}{2x^2 + 1} > 1.99 \quad \text{when} \quad x > 250.4 \]

CSI:

I am assuming that the \( K \) air is cooler than body temperature.

\[ T' (t) = k (T(t) - T_s) \]

\[ \uparrow \]

constant of proportionality

This is known as Newton's Law of Cooling.
\[ a) \lim_{h \to 0} \frac{(0+h)^{2/3} - 0^{2/3}}{h} = \lim_{h \to 0} \frac{1}{\sqrt[3]{h}} \quad \text{DNE} \]

\[ b) \lim_{x \to a} \frac{x^{2/3} - a^{2/3}}{x-a} = \lim_{x \to a} \frac{x^{2/3} - a^{2/3}}{x-a} \cdot \frac{(x^{2/3} + a^{2/3} + a^{2/3})}{(x^{2/3} + a^{2/3} + a^{2/3})} \]

\[ = \lim_{x \to a} \frac{x^{2/3} - a^{2/3}}{x-a} \cdot \frac{x^{2/3} + a^{2/3} + a^{2/3}}{x^{4/3} + a^{2/3}x^{2/3} + a^{4/3}} \]

\[ = \lim_{x \to a} \frac{x^{2} - a^{2}}{(x-a)(x^{4/3} + a^{2/3}x^{2/3} + a^{4/3})} \]

\[ = \lim_{x \to a} \frac{(x+a)(x-a)}{(x-a)(x^{4/3} + a^{2/3}x^{2/3} + a^{4/3})} \]

\[ = \frac{a + a}{3(a^{4/3})} = \frac{2a}{3a^{4/3}} = \frac{2}{3} a^{-1/3} \]

\[ \text{I dare you to solve this by def'n of the derivative!} \]

\[ 2a \]

\[ \text{I found my mistake!} \]

\[ \text{(Even I make mistakes: We know from yay!)} \]

\[ c) \text{since when } a \to 0, \quad \frac{2}{3} a^{-1/3} \to \infty \text{ the slope of the tangent line is infinitely steep: this means } f \text{ has a vertical tangent line.} \]