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MATH 141 – EXAM 1
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Directions: Answer every question. Show all work and justify your answers.

1. (a) [10 points] The position of a particle is given by the equation

\[ s(t) = -5t^2 + 2t + 3 \]

Find the average velocity of the particle from \( t = 1 \) to \( t = 2 \).

\[
\frac{s(2) - s(1)}{2 - 1} = \frac{-20 + 4 + 3 - (-5 + 2 + 3)}{1} = -13
\]

(b) [10 points] Explain in 1–3 sentences how you would estimate the instantaneous velocity of the particle at \( t = 1 \). You do not need to do the calculation.

Choose two points: \((1,0)\) and a point on graph very near \((1,0)\) and find the slope of the secant line connecting them. As the 2nd point on the graph gets nearer to the point \((1,0)\) the slope of the secant line gets closer to the value of the instantaneous velocity.

2. (a) [10 points] How do you show that a function \( f(x) \) is continuous at \( a \)?

1. Verify that \( f(a) \) exists.
2. Verify that \( \lim_{x \to a} f(x) \) exists.
3. Verify that \( f(a) = \lim_{x \to a} f(x) \).

(b) [10 points] Determine where the following function is continuous:

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x < 1 \\
  \ln x + 1 & \text{if } 1 \leq x < e \\
  x & \text{if } x \geq e 
\end{cases} \]

Need to check \( x=1 \) and \( x=e \).

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) = 1 \\
\lim_{x \to e^-} f(x) = 2 \quad \lim_{x \to e^+} f(x) = e \quad \text{not cts. at } e \\
\]

\( f(x) \) is cts. for \( (-\infty, e) \cup (e, \infty) \).
3. Consider the following parametric equations:

\[ x = t + 2 \]
\[ y = -t - 1 \]

(a) [10 points] Find a Cartesian equation of the curve.

\[ t = x - 2 \]
\[ y = -(x - 2) - 1 \]
\[ y = -x + 1 \]

(b) [10 points] Sketch the graph of the parametric curve, using an arrow to indicate the direction in which the curve is traced as \( t \) increases.

(c) [5 points] Find a different set of parametric equations that describe the same curve.

\[ x = t \]
\[ y = -t + 1 \]

(\textit{infinitely many right answers.})

4. [10 points] Find an expression for \( \delta \) so that for any \( \epsilon > 0 \), \( |4x - 8| < \epsilon \) whenever \( |x - 2| < \delta \).

\[ 4 \left| x - 2 \right| < \epsilon \]
\[ |x - 2| < \frac{\epsilon}{4} \]
\[ \omega^+ \delta = \frac{\epsilon}{4} \]
5. [5 points each] Evaluate each limit algebraically (not from a graph or a table), if it exists. If it does not exist, just write DNE.

(a) \( \lim_{x \to 0} \sqrt{x} \) \( \quad \) DNE \( \quad \) (\( \lim_{x \to 0^-} \sqrt{x} \) is undefined)

(b) \( \lim_{x \to 0} \frac{1}{x^3} \) \( \quad \) DNE

(c) \( \lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{x(x-4)}{(x+1)(x-4)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5} \)

(d) \( \lim_{x \to -3} \frac{x^2 - 9}{x + 3} = \lim_{x \to -3} \frac{(x+3)(x-3)}{(x+3)} = -6 \)

(e) \( \lim_{x \to 0} \sin(1/x) \) \( \quad \) DNE

(f) \( \lim_{x \to 4} \frac{3x - 2}{x^2 + 1} = \frac{3(4) - 2}{16 + 1} = \frac{10}{17} \)

6. (a) [10 points] Precisely state the Intermediate Value Theorem.

Suppose \( f \) is cts. on closed interval \([a, b]\) and let \( N \) be any number between \( f(a) \) and \( f(b) \) where \( f(a) \neq f(b) \).
Then there exists a number \( c \) in \((a, b)\) such that \( f(c) = N \)

(b) [10 points] Give an example that shows why the assumption of continuity is essential in the IVT. Be sure you explain what you are showing.

\[ f \] \( f(a) < 0 \) and \( f(b) > 0 \). But because this function is not cts. there is no \( c \in (a, b) \) where \( f(c) = 0 \).

(Answers may vary.)