(1) Consider the linear system

\[ \begin{align*}
0.03x_1 + 58.9x_2 &= 59.2 \\
5.31x_1 - 6.10x_2 &= 47.0.
\end{align*} \]

(a) Find the solution (in exact arithmetic)
(b) Solve the system using 3-digit arithmetic with rounding.
(c) Solve the system using 3-digit arithmetic with rounding and scaled partial pivoting.

(2) Given

\[ A = \begin{bmatrix}
25 & 0 & 0 & 0 & 1 \\
0 & 27 & 4 & 3 & 2 \\
0 & 54 & 58 & 0 & 0 \\
0 & 108 & 116 & 2 & 0 \\
100 & 0 & 0 & 0 & 24
\end{bmatrix} \]

(a) Determine a unit lower triangular matrix \( M \) and an upper triangular matrix \( U \) such that \( MA = U \).
(b) What is the determinant of \( A \)?

(3) A real \( n \times n \) matrix \( Q \) is said to be orthogonal if \( Q^t Q = I \). Show that the determinant of an orthogonal matrix is either \(-1\) or \(1\).

(4) Apply the Doolittle direct factorization technique (not Gauss elimination!) to the matrix

\[ A = \begin{bmatrix}
2 & -1 & 0 \\
-1 & 6 & -2 \\
4 & -3 & 8
\end{bmatrix}. \]

Show your calculations.

(5) Suppose \( A \) and \( B \) are both positive definite \( n \times n \) matrices. State whether the following statements are true or false:

(a) \( A + B \) is positive definite,
(b) \( A - B \) is positive definite,
(c) \( A^T \) is positive definite,
(d) \( A^2 \) is positive definite,
(e) \( A^{27} \) is positive definite.

Fully justify your claims either by providing a proof or a counterexample.

(6) Prove the following:

(a) The product of two lower triangular matrices is lower triangular. State and prove, using the above result, a similar result for the upper triangular case.
(b) The inverse of a lower triangular matrix is lower triangular. If in addition the diagonal elements are unit, then the diagonal elements of the inverse are also unit. State and prove, using the above result, a similar result for the upper triangular case.