Math 141: Test 3
Fall 2015

Directions: You must show all your work in order to receive full credit. Please write clearly and legibly and organize your work for maximum possible points. Points will be deducted for poor notation and sloppy, unreadable work. Calculators are permitted, but no points will be awarded for answers read only off the calculator with no supporting work shown.

Remember to take your time and read each question very carefully. Be sure you know exactly what each question is asking before you answer it. This is your time to show off what you know. Good luck!

1. (5 points) True/False Questions: Circle T or F in the following statements:
   
   (a) T F If $f$ is concave up and $f'$ changes sign at $x = c$, then $f''$ changes sign from negative to positive at $x = c$.
   
   (b) T F If $f'(x)$ exists and is nonzero for all $x$, then $f(1) \neq f(0)$.
   
   (c) T F If $f'(c) = 0$, then $f$ has either a local max or local min at $x = c$.
   
   (d) T F A function that is concave up on $(-\infty, \infty)$ can have no maximum value.
   
   (e) T F If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

2. (5 points) Use the graph of $f(x)$ given below to answer the following questions.

   ![Graph of f(x)]

   (a) What are the critical points of $f(x)$?

   (b) Which of the following are true? (Circle all that apply.)
   
   (i) $f'(3) < f''(4)$  
   (ii) $f'(3) > f''(4)$  
   (iii) $f''(4) > f'(6)$

   (c) What are the extreme values of $f$ over the interval $[0,9]$?
3. (13 points) Use the first and second derivative of \( f \) given below to answer the following questions. \textbf{Note:} The domain of \( f \) is all real numbers.

\[
    f'(x) = \frac{1 - x^2}{(1 + x^2)^2} \quad f''(x) = \frac{2x(x^2 - 3)}{(x^2 + 1)^3}
\]

(a) Determine the intervals where \( f \) is increasing/decreasing. Write your answer in interval notation.

(b) At what \( x \) value(s) does \( f \) have a local maximum?

(c) At what \( x \) value(s) does \( f \) have a local minimum?

(d) Determine the intervals of concavity of \( f \). Write your answer in interval notation.

(e) At what \( x \) value(s) does \( f \) have an inflection point?
(f) Sketch a graph of $f$ using what you determined on the previous page and the additional information that at $x = 0$, $f$ crosses the $x$-axis for the only time, $\lim_{x \to \infty} f(x) = 0$, and $\lim_{x \to -\infty} f(x) = 0$.

4. (3 points) Match the description of $f$ with the graph of its derivative $f'$.

(a) $f$ is increasing and concave up ____________
(b) $f$ is decreasing and concave up ____________
(c) $f$ is increasing and concave down ____________

5. (5 points) Find the extreme values of $g(x) = x + \frac{1}{x}$ on the interval $[0.2, 4]$ (using calculus, not the graph of the function.)
6. (10 points) Compute the following limits. Give exact answers only or state that the limit does not exist. (Be diligent with your limit notation for full credit.)

(a) \( \lim_{x \to 4} \frac{\sqrt{5} + x - 3}{x^2 - 3x - 4} \)

(b) \( \lim_{x \to 0} (\csc x - \cot x) \)

(c) \( \lim_{x \to 0} (1 - 2x)^{1/x} \)
7. (8 points) Marilyn the magician is performing a magic trick when something goes wrong. During her show, a loose balloon leaves the ground 500 feet away from Marilyn and rises vertically at a rate of 125 feet per minute. At what rate is the angle between Marilyn’s line of sight and the horizontal increasing when the balloon is 250 feet off the ground? (Remember to show all necessary steps and do not forget units. You will not get credit for just an answer!)

8. (1 point) You’ve made it to the finish line! Relax now and for a free point, tell me what your favorite thing about calculus is so far.