MATH 141, Unit III Test (Fall 2015)

You will have **fifty** minutes to complete this exam. Please read each question carefully. Only write on the paper that I provide you. If you need additional space to write, you may write on the back of the test paper, but please be sure to let me know when you write on the back. In order to receive credit for any problem, you **must show your work**! The test consists of 10 questions (for a total of 100 points). You may use an approved calculator during the exam. You may also leave your answers in exact form; however, you are expected to know and evaluate common values of functions like \( \sin(x) \), \( \cos(x) \), and \( \ln(x) \).

Simply using your calculator to find the answer to any of these questions will not receive any credit. You need to support all of your answers with justifiable steps.

During the test, the following activities will be considered cheating and result in the grade of zero:

1. any communication with another student while you or the other student is taking the exam
2. looking at or using your cell phone
3. looking at or using a computer
4. looking at or using your book
5. looking at or using your notes
6. using an unapproved calculator
7. using other paper
8. anything else that could be considered cheating.

If you have a question, please raise your hand and I will come over to you. I will answer questions about the statement of a problem or the form an answer should be in. I will not answer questions such as “How do I start Problem #7?”

There is more than one version of this test. If you have correct answers from another version of the test on your test, you may receive a zero for the test depending on the severity of the offense.

**Good Luck! I hope that each of you does well on this test.**
1. **[10 pts]** Suppose that an everywhere differentiable function $f$ has critical points at $x = -6$, $x = 0$, and $x = 3$. Furthermore, suppose we know that $f'(-7) = 2$, $f'(-4) = -3$, $f'(2) = 3$, and $f'(5) = 2$. Draw a sign chart for the function $f$ and for each critical point, determine if it corresponds to a relative maximum, a relative minimum, or neither.

2. **[8 pts]** Suppose that an everywhere differentiable function $f$ has critical points at $x = 1$, $x = 7$ and $x = 10$. Furthermore, suppose that $f''(1) = 3$, $f''(7) = -2$ and $f''(10) = 0$. For each critical point, determine if it corresponds to a relative maximum, a relative minimum, or if we do not have enough information to decide.

3. **[4 pts]** Does every continuous function on the interval $(-3, 4)$ have an absolute maximum and a absolute minimum? Justify your answer.
4. [15 pts] Water flows out of a funnel shaped like a right circular cone of radius $\sqrt{3}$ m and of height $\sqrt{\pi}$ m at a rate of $36m^3/s$. Find how fast the depth of water in the funnel is changing when when the height is equal to 2. **Hint:** The volume of a right circular cone is given by $V = \frac{\pi}{3}r^2h$. 
5. [14 pts] Find the absolute maximum and the absolute minimum of the function \( f(x) = 3x^4e^x \) on the interval \([-5, -1]\).

6. [13 pts] Find the intervals on which the function \( f(x) = \ln(1 + x^2) \) is concave up and concave down.
7. [8 pts] Evaluate

\[ \lim_{x \to 0} \frac{\cos(2x) - 1}{(x + 1)^2} \]

8. [9 pts] Suppose that the graph of a function \( f \) is given below. Answer the following questions

(a) What are critical points of \( f \)?

(b) What is the absolute maximum of \( f \) on the interval \([-1, 4]\)?

(c) Over what interval or intervals is \( f'(x) \) positive?

(d) Is \( f''(-2) \) positive or negative?
9. [8 pts] Evaluate

\[ \lim_{x \to 0} \frac{e^{2x} - 2x - 1}{\cos(5x) - 1} \]

10. [12 pts] On the coordinate grid below, sketch the graph of a continuous function \( f \) which satisfies the following properties:

(a) \( f(0) = 2 \)
(b) The absolute maximum of \( f \) on the interval \([0, 6]\) occurs at \( x = 2 \)
(c) \( f' < 0 \) on \((2, 4)\)
(d) \( f'' < 0 \) on \((3, 5)\)
(e) \( f \) has a local minimum at \( x = 6 \)
(f) \( \lim_{x \to \infty} f(x) = 3 \)