1. (5 points) If \( f(x) \) is concave up, then is \( f'(x) \) increasing, decreasing, or possibly either?

2. (5 points) If \( f'(c) = 0 \) and \( f''(c) < 0 \), what conclusions can you draw? Explain.

3. (5 points) Consider \( f(x) = e^x \) and \( g(x) = x^{1,000,000} \). Can we compute \( \lim_{x \to \infty} \frac{f(x)}{g(x)} \)? If so, compute the limit. Otherwise, explain why we cannot compute the limit.

4. (5 points) If \( f(x) \) is continuous on \([0, 1]\), then can we find the extreme values by just considering points where \( f'(x) = 0 \) and the endpoints? Explain.

5. (9 points) Find the extreme values of \( f(x) = 3x^4 - 4x^3 \) on \([-1, 2]\).
6. (9 points) Find any inflection points and intervals of concavity of $f(x)$ given

$$f(x) = \frac{4x}{x^2 + 3} \quad f'(x) = \frac{4(3 - x^2)}{(x^2 + 3)^2} \quad f''(x) = \frac{8x(x^2 - 9)}{(x^2 + 3)^3}.$$ 

7. (9 points) Find where $f(x) = x^2 + x^{-2}$ is increasing or decreasing and find any local extrema.
8. (10 points) Use the following information to answer parts (a), (b), and (c):

\[ f(-3) = 1, \ f(-2) = 0, \ f(0) = 2, \ f(2) = -3, \ f(3) = -2, \ \lim_{x \to -\infty} f(x) = 2, \ \lim_{x \to \infty} f(x) = -1 \]

\[ f' \quad - \ | \ + \ | \ - \ | \ + \quad f'' \quad - \ | \ + \ | \ + \ | \ - \]

(a) Find and label the local extrema for \( f(x) \).

(b) Find any inflection points of \( f(x) \).

(c) Graph \( f(x) \) in Figure 1.

![Figure 1: Graph of \( f(x) \) for Question 8](image_url)
9. (5 points) The graph of $f(x)$ is given in Figure 2
(a) What are the critical points of $f(x)$?

(b) Find the extreme values of $f(x)$ on $[0, 2]$.

10. (5 points) The graph of $g'(x)$ is given in Figure 2
(a) Where will $g(x)$ have local maximums?

(b) Where will $g(x)$ be concave up?

Compute the following limits. (18 points)

11. $\lim_{x \to \infty} \ln \left( x^{\frac{1}{x}} \right)$
12. \( \lim_{x \to 2^+} \frac{8}{x^2 - 4} - \frac{x}{x - 2} \)

13. \( \lim_{x \to 0} \frac{\cos(2x) - 1}{\sin(5x)} \)
14. **(15 points)** Jon is trying to scale a wall made of ice. He decides to use a 25 ft ladder. He leans his ladder against the wall and the distance from the bottom of the wall to the bottom of the ladder is 1 ft. The wall is not the only icy thing around, in fact the ground is covered in ice as well. Jon starts to climb the ladder. Much to the embarrassment of Jon, once he has climbed halfway up, the bottom of the ladder starts to slide away from the wall at a rate of 3 \( \frac{ft}{s} \). How quickly is the top of the ladder moving when the bottom of the ladder is 7 ft away from the bottom of the wall?