(10 pts)

1) It costs the ALBUNDY Auto Company $8000 to produce each car with fixed costs of $20,000 a week. The company’s price function is \( p = 22,000 - 70q \), where \( p \) is the price at which exactly \( q \) cars will be sold.

   a) The cost equation, \( C(q) \).
   \[
   C(q) = 20000 + 8000q
   \]

   b) The revenue equation, \( R(q) \).
   \[
   R(q) = pq = (22000 - 70q)q = 22000q - 70q^2
   \]

   c) The profit equation, \( \pi(q) \).
   \[
   \pi(q) = (22000q - 70q^2) - (20000 + 8000q) = 14000q - 70q^2 - 20000
   \]

   The production level that will maximize profit.

   \[
   \pi'(q) = -140q + 14000 = 0 \Rightarrow 140q = 14000 \Rightarrow q = 100
   \]

   Now, \( \pi''(q) = -140 < 0 \), so we have a maximum!

2) (10 pts) Find the equation (\( y = mx + b \) form) of the line tangent to \( f(x) = \ln\left(\frac{5x + 1}{x}\right) \) at \( x = 0 \).

   First we need the point on \( f(x) \): \( f(0) = \ln(0+1) = \ln(1) = 0 \). So \( (0,0) \) is the point.

   Next we need the slope of the line tangent:
   \[
   f'(x) = \frac{1}{5x+1}\bigg|_{5} = \frac{5}{5x+1}. \]

   Then the slope of the line tangent to \( f(x) \) at \( (0,0) \) is \( f'(0) = \frac{5}{0+1} = 5 \). So
   We have \( y - 0 = 5(x - 0) \Rightarrow y = 5x \).
3) Differentiate, show all work. Do not simplify!  (4 pts each)

a) \( y = 3^{\ln x} \)
\[
y' = 3^{\ln x} \left( \frac{1}{x} \right) \ln 3
\]

b) \( y = 10^{x^3+8x} \)
\[
y' = 10^{x^3+8x} \left[ 3x^2 + 8 \right] \ln 10
\]

c) \( y = x^\pi \pi^x \)
\[
y' = x^\pi \left[ \pi^x \ln \pi \right] + \pi^x \left[ \pi x^{\pi-1} \right]
\]

d) \( y = (x^2 + \ln x)^{13} \)
\[
y' = 13(x^2 + \ln x)^{12} \left[ 2x + \frac{1}{x} \right]
\]

e) \( y = \frac{\sqrt{x}}{e^{5x}} \)
\[
y' = \frac{e^{5x} \left[ \frac{1}{2} x^{-\frac{1}{2}} \right] - \sqrt{x} \left[ 5e^{5x} \right]}{(e^{5x})^2}
\]
4) **(4 points each)** For \( f(x) = 1 - 9x + 6x^2 - x^3 \), determine:

a) Critical value(s) 

\[
f'(x) = -9 + 12x - 3x^2 = -3(x - 3)(x - 1) = 0
\]

\( x = 1, 3 \)

b) Critical point(s) 

\( (1, -3) \) & \( (3, 1) \)

c) Intervals where \( f(x) \) is: 

Increasing: \((1, 3)\)

Decreasing: \((-\infty, -1) \cup (3, \infty)\)

d) Intervals where \( f(x) \) is: 

Concave Up: \((-\infty, 2)\)

Concave Down: \((2, \infty)\)

\[
f' \quad + \quad 0 \quad -
\]

\[
f''
\]

\( f''\)  

\[
\begin{array}{c}
- \\
0 \\
+ \\
0 \\
- \\
2
\end{array}
\]

e) Local Maximums and/or Local Minimums. 

\( f''(1) > 0 \) so \((1, -3)\) relative minimum

\( f''(x) < 0 \) so \((3, 1)\) Relative maximum.

f) Point(s) of inflection (if any):

\( f''(1) > 0 \) so \((1, -3)\) relative minimum

\( f''(x) < 0 \) so \((3, 1)\) Relative maximum.
5) **(20 points)** For $T(x) = (2x+1)^8$, $-1 \leq x \leq 1$ determine the global maximum and global minimum values of the function.

$$T'(x) = 8(2x+1)^7[2] = 16(2x+1)^7 = 0$$, so $x = -\frac{1}{2}$. So we need $T(-1) = 1$

$$T\left(-\frac{1}{2}\right) = 0$$, Minimum

$T(1) = 3^8$, Maximum.

6) **(20 pts)** The total cost of producing $q$ units of an item is $C(q) = 400q + 10q^2 - 0.1q^3$.

a) Determine the average cost function, $a(q)$.

$$a(q) = \frac{C(q)}{q} = 400 + 10q - 0.1q^2$$

b) Determine analytically the exact value of $q$ at which the average cost is minimized.

$$a'(q) = 10 - 0.2q = 0$$

$$10 = 0.2q \quad \Rightarrow \quad q = 50$$

Note: $a'(q) = -0.2 < 0$, so we indeed have a **Maximum**, not the required minimum!!