Math 125  Exam 1  Spring 05  Name
No Work-No Credit  SS#

For numerical answers use the full display of digits from the calculator. If calculating $ round to the nearest cent.

1) Use this graph of \( f(x) \) to evaluate the following limits.

a) \( \lim_{{x \to 2^-}} f(x) = 2 \)  
b) \( \lim_{{x \to 2^+}} f(x) = 3 \)  
c) \( \lim_{{x \to 2^-}} g(x) = \text{DNE} \)

d) \( \lim_{{x \to 3^-}} f(x) = 4 \)  
e) \( f(2) = 3 \)  
f) \( f(3) = \text{DNE} \)

2) Given \( J(x) = \frac{1}{x-1} + \sqrt{x} \) determine the domain of \( J(x) \), \( D_J \).

For \( \frac{1}{x-1} \) we have that \( x \neq 1 \) and for \( \sqrt{x} \) we have \( x \geq 0 \). These two sets will overlap for \( D_J = [0,1) \cup (1,\infty) \).

3) Determine the vertex of the quadratic function \( Q(x) = 2x^2 - 3.2x + 5 \).

For the vertex \( x = \frac{-b}{2a} = \frac{3.2}{4} = 0.8 \) . Then the y-value is \( Q(0.8) = 3.72 \), so \( (0.8,3.72) \) is the vertex.

4) Given \( f(x) = \frac{1}{x+2} \) and \( g(x) = 1-x \) evaluate and simplify:

a) \( (f \circ g)(x) = f(g(x)) = f(1-x) = \frac{1}{(1-x)+2} = \frac{1}{3-x} \)

b) \( (g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+2}\right) = 1 - \frac{1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+1}{x+2} \)
5) State the formal definition of the derivative of f(x):

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

b) K(x) is continuous at x = a if \( \lim_{x \to a} f(x) = f(a) \).

6) Evaluate:  

a) \( \lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x+1)} \lim_{x \to 2} \frac{x+2}{x+1} = \frac{4}{3} \).

7) Given \( f(x) = \frac{x-2}{x+1} \) determine the average rate of change from \( x = -2 \) to \( x = 0 \).

Here \( h = 2 \) so we have \( \frac{f(0) - f(-2)}{2} = \frac{-2-4}{2} = -3 \).

8) Use the formal definition of the derivative to determine \( f'(x) \) if \( f(x) = x - 2x^2 \).

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\left[ (x+h) - 2(x+h)^2 \right] - \left[ x - 2x^2 \right]}{h} \\
&= \lim_{h \to 0} \frac{\left[ x + h - 2x^2 - 4xh - 2h^2 \right] - \left[ x - 2x^2 \right]}{h} \\
&= \lim_{h \to 0} \frac{-4xh - 2h^2}{h} \\
&= \lim_{h \to 0} \frac{h(1-4x-2h)}{h} = \lim_{h \to 0} (1-4x-2h) = 1 - 4x.
\end{align*}
\]