MATH 435- Problems on harmonic functions-due 4/10/2002

1. Solve $\Delta u = 0$ in the spherical shell $0 < a < r < b$ in $\mathbb{R}^3$ with boundary conditions $u = A$ on $r = a$, $u = B$ on $r = b$, where $A$ and $B$ are constants (look for a solution depending only on $r$).

2. Solve $\Delta u = 1$ in the annulus $a < r < b$ in $\mathbb{R}^2$, with $u$ vanishing on the boundary (that is, $u = 0$ at $r = a$ and $r = b$).

3. Show that there are no solutions of:
$$\Delta u = f \text{ in } D, \quad \frac{\partial u}{\partial n} = g \text{ on } \partial D,$$
for $D \subset \mathbb{R}^3$ bounded, unless:
$$\int_D f \, dv = \int_{\partial D} g \, dA.$$  
(Hint: divergence theorem).

4. Suppose that $u$ is a harmonic function in the disk $D = \{r < 2\} \subset \mathbb{R}^2$ and that $u = 3 + 5 \sin(7\theta)$ for $r = 2$. Without finding the solution, (i) find the maximum value of $u$ in $D$; (ii) find the value of $u$ at the origin.

5. Solve $\Delta u = 0$ in the disk $\{r < R\} \subset \mathbb{R}^2$, with boundary condition
$$u = 2 + 3 \cos(2\theta) \text{ on } r = R.$$  

6. Solve $\Delta u = 0$ in the exterior $\{r > R\}$ of a disk of radius $R$ in $\mathbb{R}^2$, with the boundary condition $u = 1 - 2\sin(5\theta)$ on $r = R$, and the condition at infinity that $u$ be bounded as $r \to \infty$. Without this condition, is the solution unique?