(39a) \[(1, 1, 1, 1, ... \cdot (0, 1, 2, 3, 4, 5, \ldots) =

(1, 1+1, 1+1+1, 1+1+1+1, \ldots = (0, 1, 2+1, 3+2+1, \ldots = (0, 1, 3, 6, 10, 15, \ldots)

The entry \((ab)_{30}\) is \(29+28+\ldots+2+1+0 = \frac{1}{2} \cdot 29 \cdot 30 = 435\)

39(b): The sum being defined as \((a+b)_n := a_n + b_n\),
the addition axioms are proved as they are proved for the
direct sum of rings, namely:

\((C+)\) \(a+b = b+a\) because for all \(n\)
\[(a+b)_n = a_n + b_n \in b_n + a_n = (b+a)_n\]

\((A+)\) \[(a+b) + c)_n = (a+b)_n + c_n = (a_n + b_n) + c_n \uparrow = a_n + (b_n + c_n) = a_n + (b+c)_n\]

\((N+)\) The sequence \((0, 0, 0, \ldots)\) consists of all zeros only
is the additive identity.

\((hr+)\) The negative of \((a_0, a_1, a_2, a_3, \ldots)\) is \((-a_0, -a_1, -a_2, -a_3, \ldots)\)

\((C0)\) \[(ab)_n = \sum_{i=0}^{n} a_i b_{n-i} \quad (ba)_n = \sum_{j=0}^{n} b_j a_{n-j}\]

We must show that these two are equal.

\[\sum_{i=0}^{n} a_i b_{n-i} = \sum_{i=0}^{n} b_{n-i} a_i = \sum_{j=0}^{n} b_j a_{n-j}\]

\((C0)\) in \(R\) uses \((C+)\) and \((A+)\) in \(R\).