28. \[ x = \sqrt{R^2 - z^2} \]
\[
\frac{dx}{dz} = \frac{-2z}{2\sqrt{R^2 - z^2}}
\]
\[
cap \text{ area} = \int_{R \cos \theta}^{R} 2\pi \sqrt{R^2 - z^2} \sqrt{1 + \frac{z^2}{R^2 - z^2}} \, dz
\]
\[
= 2\pi \int_{R \cos \theta}^{R} \frac{R}{R \cos \theta} \, dz = 2\pi R^2 \left(1 - \cos \theta\right)
\]
Note: For very small \( \theta \), \( \cos \theta \approx 1 - \frac{1}{2} \theta^2 \), and we obtain the approximation \( \pi (R \theta)^2 \), with \( R \theta \) indeed the radius measured on the surface. (Think of \( R = 6,370 \text{ km} \), \( \theta = 0.001 \), \( R \theta = 6.37 \text{ km} \). You wouldn't take the curvature of the Earth into account when calculating the area of a circle of 6.37 km.)

29. \[ vol = \pi \int \left[ (R^2 - y^2) - r^2 \right] \, dy = \]
\[
= \pi \int_{-r}^{r} \left[ (R^2 - r^2) - \frac{1}{3} y^3 \right] \sqrt{R^2 - r^2} \, dy
\]
\[
= \frac{4\pi}{3} (R^2 - r^2)^{3/2}
\]
This result is a bit remarkable: You don't need \( R, r \) separately; to get the volume, it suffices to know this length: