You can use any written resources, but please work alone.

8. **Properties of the Condition Number.** For any subordinate norm $|| \cdot ||$ on $\mathbb{C}^{n \times n}$ show that the following properties are true for $\kappa(A) = ||A|| \cdot ||A^{-1}||$:

(a) $\kappa(A) \geq 1$.

(b) $\kappa(\alpha A) = \kappa(A)$ if $\alpha \neq 0$.

(c) $\kappa(A \cdot B) \leq \kappa(A) \kappa(B)$.

9. Let $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$. Consider the following algorithm for computing the dot product of $x$ and $y$:

\[
\text{dp} = 0 \\
\text{for } i = 1 \text{ to } m \\
\quad \text{dp} = \text{dp} + x(i) \cdot y(i) \\
\text{end}
\]

(a) In exact arithmetic, does this method produce the correct answer? If not, construct a bound on the error.

(b) Study the propagation of roundoff error by this algorithm. Construct a bound on the difference between the exact solution and the computed solution in terms of $\epsilon_{\text{mach}}$ and other terms.

(c) Count the number of flops for this algorithm.

10. Let

\[
A = \begin{pmatrix}
1 & 1 + \delta & 1 \\
1 & 1 & 1 + \delta \\
1 & 1 & 1
\end{pmatrix}.
\]

Compute, by hand, the $QR$ factorization of $A$ using (a) Classical Gram-Schmidt and (b) Modified Gram-Schmidt. In exact arithmetic they will give the same answer, but in FPA they won’t. Identify, if possible, where the methods differ (for small $\delta$).