20. Find all \( x \in \mathbb{R} \) for which the inequality \( \frac{x}{x+2} < 3 \) is true.

21. Find all \( x \in \mathbb{R} \) for which the inequality \( \frac{x-1}{2x-1} < -1 \) is true.

22. Find all \( x \in \mathbb{R} \) for which the inequality \( \frac{x-2}{2x-4} > \frac{1}{2} \) is true.

23. Prove \( a \) is positive if and only if \( -a \) is negative.

24. If \( a < b \) and \( c < 0 \), prove \( ac > bc \).

25. If \( a < b \) and \( c < d \), prove \( a + c < b + d \).

26. If \( 0 < a < b \) and \( 0 < c < d \), prove \( ac < bd \).

27. If \( a < b \), \( b \geq 0 \), and \( 0 \leq c \leq d \), prove \( ac \leq bd \).

28. If \( a < b \), \( a > 0 \), and \( c < d < 0 \), prove that \( ad > bc \).

29. Prove \(-1 < 0 < 1\).

30. If \( 0 < a < 1 \), prove \( a^2 < a \).

31. If \( a > 1 \), prove \( a < a^2 \).

32. Suppose \( a > 0 \). If \( a < b \), prove \( a^2 < b^2 \).

33. Suppose \( a > 0 \). Prove that \( a < b \) if and only if \( b > 0 \) and \( a^n < b^n \) for some \( n \in \mathbb{N} \).

34. If \( 0 < a < b \), prove \( a^{-1} > b^{-1} > 0 \).

35. If \( 0 < a < 1 \), prove \( \frac{1}{a^2} > \frac{1}{a} \).

36. If \( a > 1 \), prove \( \frac{1}{a^2} < \frac{1}{a} \).

37. If \( a \leq b \), prove \( a + c \leq b + c \) for all \( c \in \mathbb{R} \).

38. If \( a \leq b \), and \( c \geq 0 \), prove \( ac \leq bc \).

39. Prove \( ab > 0 \) implies either both \( a \) and \( b \) are positive or both \( a \) and \( b \) are negative.

40. If \( a > 0 \), prove that there is an \( \epsilon > 0 \) such that \( a > \epsilon \).

41. If \( a > 0 \), prove that there is an \( \epsilon \in (0, 1) \) such that \( \epsilon < a \).

42. Suppose \( a, b \in \mathbb{R} \). Prove that \( a < b + \epsilon \) holds for all \( \epsilon > 0 \) if and only if \( a \leq b \).

43. Prove that \( a > b - \epsilon \) holds for all \( \epsilon > 0 \) if and only if \( a \geq b \).