1. Let $V$ be the space of real-valued single variable functions. Let $W$ be the collection of all functions that have a zero at $x = 12$. In other words, $W$ is all functions such that $f(12) = 0$. Show that $W$ is a subspace of $V$.

2. Let $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ -1 & 2 & 1 \end{bmatrix}$.
   a. Does the column space of $A$ (ie. $\text{gen}\{ \vec{c}_1, \vec{c}_2, \vec{c}_3 \}$) span $\mathbb{R}^3$? Explain.
   b. Are the column vectors of $A$ linearly independent? Explain.
   c. Would these two facts coincide for any invertible matrix $A$ in $\mathbb{R}^{3\times3}$? Explain.

3. Let $B$ be in $\mathbb{R}^{m\times n}$. If $n > m$, then the null space of $B$ contains infinitely many vectors of $\mathbb{R}^n$. Explain.

4. The space $\text{gen}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ spans diagonal matrices in $\mathbb{R}^{3\times3}$. Explain.

5. Show that $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, and $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ are linearly independent in $\mathbb{R}^{2\times2}$.

6. Let $V$ be a vector space, and let $W$ be a subspace of $V$. If $W$ contains a single non-zero vector, it must contain infinitely many vectors. Explain.