We are wanting to compute \( \frac{d}{dx} \int_0^{x^2} \frac{t}{t+1} \, dt \).

The issue here is we can't compute the antiderivative of \( \frac{t}{t+1} \).

So, we need another method:

If \( x^2 \) was just an \( x \), we'd use the fund-thm, part II, letting \( u = x^2 \) partially solve this problem.

It at least gets us to the point \( \frac{d}{dx} \int_0^{u} \frac{t}{t+1} \, dt \).

Now the issue is the \( \frac{d}{dx} \). We wish it were a \( \frac{d}{du} \).

But, \( \frac{d}{dx} = \frac{du}{dx} \cdot \frac{d}{du} \) (This is just the chain rule:

\[
\frac{d}{dx} (f(u)) = f'(u) \cdot u'
\]

in other words)

Substituting, we get \( \frac{du}{dx} \cdot \frac{d}{du} \int_0^{u} \frac{t}{t+1} \, dt \).

this is just says this is \( \frac{u}{u+1} \).

(\( \text{since } u = x^2 \)).

Now we have \( 2x \cdot \frac{u}{u+1} \). Swapping for \( x \), we get

\[
2x \cdot \frac{x^2}{x^2+1} \quad \text{or simplify} \quad \frac{2x^3}{x^2+1}.
\]