**Wed Mar 23  Lecture Commentary**

**Why the 1,4,2,4,2,...,2,4,1 pattern in Simpson’s Rule always works out nicely**

Recall the formula given in class for Simpson’s Rule was

\[
\frac{\Delta x}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N) \right].
\]

It is important to keep in mind just what \(x_0, x_1, x_2, \ldots\) are when dealing with Simpson’s rule to not only make sense of why the pattern always holds, but also to make sure you are computing the correct values.

The values \(x_0, x_1, x_2, \ldots\) are the endpoints of the \(N\) subintervals that result from \([a, b]\) being chopped into \(N\) pieces of equal length. Normally, we build a region of area (ie. rectangle, trapezoid, etc.) for each single-wide subinterval. For Simpson’s rule, we build a region of area for each double-wide subinterval.

What I mean is this. Consider this picture associated with a sample \(T_3\) computation:

![Graph showing Simpson's Rule for T3 computation](image)

Note, there are three area regions, namely \([x_0, x_1]\), \([x_1, x_2]\), and \([x_2, x_3]\).

Now, consider this picture associated with a sample \(S_6\) computation:

![Graph showing Simpson's Rule for S6 computation](image)

Note, there are three area regions, namely \([x_0, x_2]\), \([x_2, x_4]\), and \([x_4, x_6]\).

As you see, in any Simpson’s Rule computation, the even-indexed \(x_i\) values are the endpoints of your area regions, and the odd-indexed \(x_i\) values are the midpoints of those area regions. Since you need an even-indexed \(x_i\) at the endpoints of each area region, a Simpson’s Rule computation only makes sense if the subscripted number on the \(S\) is even. It always will be.