**Wed Feb 3 Lecture Commentary**

**What is meant by area “completely enclosed”**

Technically, the notion at play here is what is referred to as mathematical “compactness”. A region of the $xy$-plane is compact if and only if it is closed and bounded. What that means in effect is best visualized.

Regions that are completely enclosed (ie. closed and bounded) are shaded. All other regions cut by the two (or three) curves are not completely enclosed. (In particular, all other regions extend off to infinity.)

How to know when to use the $xy$-plane and when you use the $yx$-plane

The last problem Phan did in class today was a bit of a roller coaster ride. Let me try to sort out the situation.

First, note any completely enclosed region has the same area no matter how much you rotate the picture. Imagine your phone or computer screen being rotated. The area in the enclosed region is the same. It’s really the exact same region, just from a slightly different vantage point. So, considering the $xy$-plane or the $yx$-plane (which is practically just a rotation by $90^\circ$…technically, there is also a reflection in there too…you’ll see this if you think about the remark for column four below), you will preserve the value of the desired enclosed area region(s).

Second, how do you know when to view the functions as $y = \cdots$ in the $xy$-plane and when you view them as $x = \cdots$ in the $yx$-plane? Consider the equations given in the problem Phan worked: \[
\begin{align*}
2x - y - 2 &= 0 \\
x - y^2 + 4 &= 0
\end{align*}
\] These equations turned out easier to solve for $x = \cdots$. What about these equations makes them easier to solve for $x$ than $y$? Would it have been possible to recognize that it would have been simpler to solve for $x$ from the get-go? I claim yes. The variable $x$ is only of degree 1 in both equations! But, the variable $y$ is of degree 2. It’s that simple.

Consider the equations below:

- $x^2 - y - 6 = 0$
- $2y - x = 0$
- $8 - \sqrt{x} + y = 0$
- $x - y^2 = 0$

- $y$ not of degree 1
- Solve for $x$
- Graph in $xy$-plane
- $x$ not of degree 1
- Solve for $y$
- Graph in $yx$-plane

Neither of degree 1
Harder problem

$xy$ - or $yx$-plane a choice

A note regarding the second column, the expression $(y - 1)^2$ would become a degree 2 expression. And, a note regarding the last column, in such a situation, I have become comfortable graphing the $y = \cdots$ in the $xy$-plane and then rotating my paper $90^\circ$ counterclockwise to graph the other orientation (keeping in mind the horizontal axis is now going positive to the left). It takes a little getting used to, but it’s nothing fancy really.