Wed Feb 17 Lecture Commentary

When to use disks/washers and when to use shells

Let me just start by reiterating a point made in the last commentary (and during recitation):

Function of \( x \) revolved around \( x \)-axis means disks/washers (ie. \( \pi r^2 \, dx \) or \( (\pi R^2 - \pi r^2) \, dx \)).

Function of \( x \) revolved around \( y \)-axis means shells (ie. \( 2\pi r f(x) \, dx \)).

![Diagram of a function revolved around the x-axis](image)

Similarly,

Function of \( y \) revolved around \( y \)-axis means disks/washers (ie. \( \pi r^2 \, dy \) or \( (\pi R^2 - \pi r^2) \, dy \)).

Function of \( y \) revolved around \( x \)-axis means shells (ie. \( 2\pi r f(y) \, dy \)).

![Diagram of a function revolved around the y-axis](image)

Also note:

When dealing with a function of \( x \), you look to the \( x \)-axis for your interval of interest when integrating.

When dealing with a function of \( y \), you look to the \( y \)-axis for your interval of interest when integrating.

This may seem completely natural (and indeed it is), but these fundamentals are especially important to keep in mind when (I suspect) your problems on the test may not specifically instruct you upon which method (disks/washers or shells) on every problem. So, knowing what to do in what situation is paramount, especially when work not on route to solution receives no credit on the tests (whereas showing some command in the direction of the solution can receive healthy amounts of partial credit).

The next bit, I am going to put on its own page so you can ultimately see it easier together at a glance...
Shells around lines other than an axis line

Note the formula above for \( f(x) \) rotated about the \( y \)-axis: \( 2\pi r f(x)dx \). Here, the \( r \) is for the radius of rotation.

If you rotate about the \( y \)-axis, your \( r \) will come out to be just \( x \). To see this, just insert a Riemann rectangle at a generic \( x \) value to your picture and note its distance from the \( y \)-axis.

So, what if you are rotating about a line other than the \( y \)-axis?

Say you are rotating about the line \( x = -2 \).

Here, the radius is lengthened by \( 2 \). So, \( r = x + 2 \). Note the integral expression reflecting this.

Say you are rotating about the line \( x = 1 \).

Here, the radius is shortened by \( 1 \). So, \( r = x - 1 \). This is reflected in the integral expression.

Say you are rotating about the line \( x = 10 \).

Here, the radius is \( 10 - x \). The volume integral to the right uses this.