Trig Identities: which ones we’ve used and when to use them

At the tail end of class, a student from the back row boldly inquired about the use of a formula sheet on the test. Phan was clear this is not going to be allowed. But, in an effort to maximize test prep efforts, I wanted to take a moment to compile a list of the trig identities we’ve used during this chapter as well as when each tends to get used.

**Trig derivatives**

- \( \frac{d}{d\theta}(\sin \theta) = \cos \theta \) ...used when you let \( u = \sin \theta \) in an integral involving a product of powers of sine and cosine, where the power of cosine has been reduced to 1 (eg. \( \int \cos \theta (1 - \sin^2 \theta) d\theta \)).

- \( \frac{d}{d\theta}(\cos \theta) = -\sin \theta \) ...similar to above, but with \( u = \cos \theta \) when the power of sine has been reduced to 1 (eg. \( \int \cos^2 \theta (1 - \cos^2 \theta)^2 \sin \theta \cdot d\theta \)).

- \( \frac{d}{d\theta}(\tan \theta) = \sec^2 \theta \) ...used when you let \( u = \tan \theta \) in an integral involving a product of powers of sine and cosine, where the power of secant has been reduced to 2 (eg. \( \int \tan^4 \theta \cdot \sec^2 \theta \cdot d\theta \)).

- \( \frac{d}{d\theta}(\sec \theta) = \sec \theta \cdot \tan \theta \) ...similar to above, but with \( u = \sec \theta \) when the power of tangent has been reduced to 1 (eg. \( \int (1 - \sec^2 \theta) \sec^3 \theta \cdot \tan \theta \cdot d\theta \)).

**Trig integrals**

- \( \int \sec \theta \cdot d\theta = \ln |\sec \theta + \tan \theta| + C \) ...crops up occasionally in trig integral computations. Phan has done two or three problems in class where integrating secant was necessary.

- \( \int \csc \theta \cdot d\theta = \ln |\csc \theta - \cot \theta| + C \) ...also crops up occasionally during trig integrals. Phan did no problems involving integrating cosecant in class, but it cropped up in a couple book problems.

**Pythagorean identities**

- \( \cos^2 \theta + \sin^2 \theta = 1 \) ...this follows from knowledge of the fact that the unit circle has equation of \( x^2 + y^2 = 1 \) and that points on the unit circle are of the form \((x, y) = (\cos \theta, \sin \theta)\). This is often used with integrals involving a product of powers of sine and cosine to reduce a power of cosine or sine by 2 (eg. \( \int \cos^3 \theta \cdot d\theta = \int (1 - \sin^2 \theta) \cos \theta \cdot d\theta \) ...now, you can let \( u = \sin \theta \) and solve).

- \( 1 + \tan^2 \theta = \sec^2 \theta \) ...this follows from dividing the terms on both sides of the formula above by \( \cos^2 \theta \). It is often used with integrals involving a product of powers of secant and tangent to reduce a power of secant or tangent by 2 (eg. \( \int \tan^2 \theta \cdot \sec^4 \theta \cdot d\theta = \int \tan^2 \theta (1 + \tan^2 \theta) \sec^2 \theta \cdot d\theta \) ...now, you can let \( u = \tan \theta \) and solve).
Double/Half angle identities

- $\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$...this is helpful when working with integrals that are products of powers of sine and cosine, specifically when both the powers of sine and cosine are even. In effect, this reduces a power of sine or cosine by 1, getting you to the easier odd-powered case (eg. $\int \sin^2 \theta \cdot d\theta = \frac{1}{2} \int (1 - \cos 2\theta) \cdot d\theta$).

- $\cos^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$...similar to above.

Hyperbolic trig integrals

- Phan only ever assigned problems in the most basic of forms with these. Having only a basic knowledge of these identities should be sufficient to your being able to work whatever problem Phan would expect of you.

- $\int \sinh \theta \cdot d\theta = \cosh \theta + C$

- $\int \cosh \theta \cdot d\theta = \sinh \theta + C$

- $\int \sec h^2 \theta \cdot d\theta = \tanh \theta + C$

- $\int \sec h \theta \cdot \tanh \theta \cdot d\theta = \sec h \theta + C$

- $\cosh^2 \theta - \sinh^2 \theta = 1$...follows from $x^2 - y^2 = 1$ being the equation of the unit hyperbola, and that points on the unit hyperbola being of the form $(x, y) = (\cosh \theta, \sinh \theta)$. 