Section 1.5 - Exponential Functions

**Definition:** A function that has a variable in an exponent is called an exponential function.

- **Examples:**

(a)

Base:

Table of Values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph:

(b)

Base:

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<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Graph:
• We can define an exponential function \( f(x) = a^x \) for any positive base \( a \). We always take the base to be positive.

• Exponential functions with bases \( a > 1 \) are used to model growth, as in populations or savings accounts.

• Exponential functions with bases \( a < 1 \) are used to model decay, as in depreciation.

• What happens if \( a = 1 \)?

**Compound Interest**

• What is interest?

• The word *compound* means that the interest is added to the account, earning more interest.

**Example:** Find the value of $5000 invested for 4 years at 10% compounded annually.

Banks always state *annual* interest rates, but the compounding may be done more than just once a year. For example, if a bank offers 8% compounded quarterly, then each quarter you get 2% (one quarter of the annual 8%), so that 2% of your money is added to the account each quarter.

**Example:** Find the value of $3000 invested for 1 year at 8% compounded quarterly.

\[
\text{For } P \text{ dollars invested at (annual) interest rate } r \text{ compounded } m \text{ times a year,}
\]
\[
\text{(Value after } t \text{ years)} = P\left(1 + \frac{r}{m}\right)^{mt}
\]
Use $m = ________$ for yearly compounding.

Use $m = ________$ for quarterly compounding.

Use $m = ________$ for monthly compounding.

Use $m = ________$ for daily compounding.

**Example:** Find the value of $\$5000$ invested for 4 years at 10\%$ compounded annually.

**Example:** Find the value of $\$3000$ invested for 1 year at 8\%$ compounded quarterly.

**Example:** Find the value of $\$2000$ invested for 3 years at 24\%$ compounded monthly.

**Depreciation by a Fixed Percentage**

Depreciation by a fixed percentage means that an object loses a fixed percentage of its value each year. Losing a percentage of value is like compound interest but with a negative interest rate. Therefore, we use the compound interest formula with $m = 1$ (since depreciation is annual) and with $r$ being negative.

**Example:** A printing press, originally worth $\$50,000$, loses 20\% of its value each year. What is its value after 4 years?
Continuous Compounding
Under continuous compounding, the interest is added to your account as it is earned, with no delay.

For \( P \) dollars invested at (annual) interest rate \( r \) compounded continuously, (Value after \( t \) years) = \( Pe^{rt} \)

Remember, the value of the number \( e \) is \( e = 2.71828... \).

**Example:** Find the value of $1000 deposited in a bank at 10% interest for 8 years compounded continuously.

\[
\text{The Function } y = e^x
\]

**Base:**

Table of Values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
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<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<tr>
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<td></td>
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<td></td>
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<td></td>
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</table>

Graph:

Note the following from the graph of \( y = e^x \):
- \( e^x \) is never zero
- \( e^x \) is positive for all values of \( x \), even when \( x \) is negative