Avoiding Common Errors

Many properties of multiplication are not applicable to the operation of addition. Make sure you avoid the common mistakes listed below.

<table>
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<tr>
<th>Correct multiplication property</th>
<th>Common error with addition</th>
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-Examples: State whether the given equation is true for all values of the variables. (Disregard any value that makes a denominator zero.)

(a)

(b)

(c)

(d)
Section 1.5 - Equations

Definition: An equation is a statement that two mathematical expressions are equal.

EQUATIONS INVOLVING A SINGLE VARIABLE
The focus in this section is solving equations involving a single variable. To solve an equation involving a single variable, we find all values of the variable that make the equation true. These values are called the solutions or roots of the equation.

In solving an equation, we are allowed to

1. add the same quantity to both sides of the equation
2. multiply both sides of the equation by the same nonzero quantity

Just remember - “Whatever you do to one side of the equation, you must do to the other.”

Linear Equations
Definition: A linear equation is an equation of the form

\[ ax + b = 0 \]

where \( a \) and \( b \) are real numbers with \( a \neq 0 \).

An approach to solving a linear equation is to put all terms that have the variable on one side of the equation and all constant terms on the other.

-Example:

Quadratic Equations
Definition: A quadratic equation is an equation of the form

\[ ax^2 + bx + c = 0 \]

where \( a, b, \) and \( c \) are real numbers with \( a \neq 0 \).

Before attempting to solve a quadratic equation, make sure the equation is written in standard form, meaning the right-hand side of the equation is 0.

Solving a Simple Quadratic Equation
A simple quadratic equation is a quadratic equation in which the coefficient of the linear term in the standard form of the equation is zero (\( b = 0 \)). It is easy to solve such an equation by isolating \( x^2 \) and taking the square root of both sides, remembering that there are two solutions to such an
equation - one positive and the other negative. That is, the solutions of the equation $x^2 = c$ are $x = \sqrt{c}$ and $x = -\sqrt{c}$.

- Examples

(a)

(b)

**Solving a Quadratic Equation by Factoring**

We can solve a quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) by factoring if $ax^2 + bx + c$ factors. The procedure is as follows:

1. Factor $ax^2 + bx + c$.

2. Set each factor equal to 0, and solve each of the resulting equations for $x$. These values will form the solution set for $ax^2 + bx + c = 0$.

- Examples

(a)

(b)

**Solving a Quadratic Equation by Completing the Square**

One might solve the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) by completing the square. This method is useful when $ax^2 + bx + c$ does not factor. The procedure is as follows:
1. Move $c$ to the right-hand side of the equation.

2. Divide both sides of the resulting equation by $a$.

3. Add to both sides of this equation the term $\left(\frac{1}{2} \frac{b}{a}\right)^2$.

4. Factor the left-hand side of the resulting equation as a perfect square trinomial.

5. Solve the resulting simple quadratic equation.

-Examples:

(a)

(b)

Solving a Quadratic Equation Using the Quadratic Formula

A simple way to solve any quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) is to use the Quadratic Formula.

**Quadratic Formula:** The solutions to $ax^2 + bx + c = 0$ (where $a \neq 0$) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Derivation of the Quadratic Formula:**
How Many Solutions Does a Quadratic Equation Have?

**Definition:** The discriminant of the general quadratic $ax^2 + bx + c = 0$ $(a \neq 0)$ is $D = b^2 - 4ac$.

- If $D > 0$, then the equation has ____________________________.
- If $D = 0$, then the equation has ____________________________.
- If $D < 0$, then the equation has ____________________________.

*Example:*
**Example:** An object thrown or fired straight upward at an initial speed of $v_0$ ft/s will reach a height of $h$ feet after $t$ seconds, where $h$ and $t$ are related by the formula

$$h = -16t^2 + v_0t$$

Suppose that a ball is thrown straight upward at an initial speed of 40 ft/s.

(a) When does the ball reach a height of 24 ft?

(b) When does it reach a height of 48 ft?

(c) What is the greatest height reached by the ball?

(d) When does the ball reach the highest point of its path?

(e) When does the ball hit the ground?

**Other Equations**

**WARNING!** When solving an equation of one of the types listed below, we may end up with one or more **extraneous solutions**, that is, potential solutions that do not satisfy the original equation. Therefore, you must always check your answers to make sure that each satisfies the original equation.
Equations Involving Fractional Expressions

**Approach:** Multiply each side of the equation by the common denominator.

- **Examples:**

(a)

(b)

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Equations Involving an \( n \)th Root

**Approach:** To eliminate the \( n \)th root, first isolate it on one side of the equal sign. Then raise both sides of the equation to the \( n \)th power.

- **Example:**

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Equations of the Quadratic Type

An equation of the form \( aw^2 + bw + c = 0 \) (\( a \neq 0 \)), where \( w \) is an algebraic expression, is an equation of **quadratic type**. We solve equations of quadratic type by substituting for the algebraic expression.

- **Examples:**

(a)
Equations Involving Absolute Value

-Example:

EQUATIONS INVOLVING SEVERAL VARIABLES

Many equations involve several variables, and it is often necessary to express one of the variables in terms of the others. To do so, we isolate this variable on one side of the equation, treating the other variables as we would numbers.

-Examples:

(a)

(b)