Section 2.9 - One-to-One Functions and Their Inverses

THE INVERSE OF A FUNCTION

Definition: Let \( f \) be a one-to-one function with domain \( A \) and range \( B \). Then its inverse function \( f^{-1} \) has domain \( B \) and range \( A \) and is defined by

\[
f^{-1}(y) = x \iff f(x) = y
\]

for any \( y \) in \( B \).

Example:

**WARNING!** Don’t mistake the -1 in \( f^{-1} \) for an exponent. That is,

\[
f^{-1} \text{ does not mean } \frac{1}{f(x)}
\]

The reciprocal \( 1/f(x) \) is written as \( (f(x))^{-1} \).

Property of Inverse Functions Let \( f \) be a one-to-one function with domain \( A \) and range \( B \). The inverse function \( f^{-1} \) satisfies the following cancellation properties.

\[
f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A
\]

\[
f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B
\]

We can use these properties to verify that two functions \( f \) and \( g \) are inverses of each other.

Examples: Show that \( f \) and \( g \) are inverses of each other.

(a)

(b)
Obtaining the Graph of $f^{-1}$ From the Graph of $f$
The graph of $f^{-1}$ is obtained by reflecting the graph of $f$ in the line $y = x$.

**How to Find the Inverse of a One-to-One Function**

1. Write $y = f(x)$.
2. Switch $x$ and $y$.
3. Solve the resulting equation for $y$.

**Examples:** Find $f^{-1}$.

(a)

(b)

(c)

Notice in each case that the inverse function “undoes” the original function - the inverse does the reverse operations in the opposite order.
Section 3.1 - Polynomial Functions and Their Graphs

A polynomial of degree $n$ is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where $n$ is a nonnegative integer and $a_n \neq 0$.

- The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the coefficients of the polynomial.
- The number $a_0$ is the constant coefficient or constant term.
- The number $a_n$, the coefficient of the highest power, is the leading coefficient, and the term $a_n x^n$ is the leading term.
- If a polynomial consists of just a single term, then it is called a monomial.

Examples:

(a)

(b)

(c)

(d)

(e)

What Does the Graph of a Polynomial Function Look Like?
- The graph of a polynomial function is always a smooth and continuous curve, meaning it has no breaks or sharp corners.
• The graph of a polynomial of **degree zero** is a ____________________________.

• The graph of a polynomial of **degree one** is a ____________________________.

• The graph of a polynomial of **degree two** is a ____________________________.

• The greater the degree of the polynomial, the more complicated its graph can be.

**Graphs of Monomials of the Form** \( P(x) = x^k \)

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<thead>
<tr>
<th>( k ) is even</th>
<th>Functions:</th>
<th>Description of graph:</th>
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The graphs of some polynomial functions can be sketched by transforming the graph of an appropriate function of the form \( f(x) = x^k \).

**Examples:**

(a)

(b)

(c)

**End Behavior**

The **end behavior** of a polynomial is a description of what happens as \( x \) becomes large in the positive or negative direction.

To describe end behavior, we use the following notation:

• \( x \to \infty \) means ___________________________________________________________.
• $x \to -\infty$ means ____________________________ \\

• $y \to \infty$ means ____________________________ \\

• $y \to -\infty$ means ____________________________

For any polynomial, the end behavior is determined by the degree of the polynomial (whether it’s even or odd) and the sign of the leading coefficient.

\[
y = P(x) \text{ has odd degree}
\]

<table>
<thead>
<tr>
<th>Leading coefficient positive</th>
<th>Leading coefficient negative</th>
</tr>
</thead>
</table>

\[
y = P(x) \text{ has even degree}
\]

| Leading coefficient positive | Leading coefficient negative |

It might be helpful to review the chart END BEHAVIOR OF POLYNOMIALS on page 260.