Section 1.5 - Equations

**WARNING!** When solving an equation of one of the types listed below, we may end up with one or more extraneous solutions, that is, potential solutions that do not satisfy the original equation. Therefore, **you must always check your answers to make sure that each satisfies the original equation.**

**Equations Involving Fractional Expressions**

**Approach:** Multiply each side of the equation by the common denominator.

- **Examples:**

(a)

(b)

**Equations Involving an n\textsuperscript{th} Root**

**Approach:** To eliminate the n\textsuperscript{th} root, first isolate it on one side of the equal sign. Then raise both sides of the equation to the n\textsuperscript{th} power.

- **Example:**

**Equations of the Quadratic Type**

An equation of the form \( aw^2 + bw + c = 0 \) (\( a \neq 0 \)), where \( w \) is an algebraic expression, is an equation of **quadratic type**. We solve equations of quadratic type by substituting for the algebraic expression.
Examples:

(a)

(b)

Equations Involving Absolute Value

-Example:

EQUATIONS INVOLVING SEVERAL VARIABLES

Many equations involve several variables, and it is often necessary to express one of the variables in terms of the others. To do so, we isolate this variable on one side of the equation, treating the other variables as we would numbers.

-Examples:

(a)

(b)
Section 1.7 - Inequalities

- An inequality looks just like an equation, except that in place of the equal sign is one of the symbols $<, >, \leq,$ or $\geq$.

- To solve an inequality that contains a variable means to find all values of the variable that make the inequality true.

- An inequality generally has infinitely many solutions, which form an interval or a union of intervals on the real line.

Rules for Inequalities

1. Adding the same quantity to each side of an inequality gives an equivalent inequality.

   -Example:

2. Subtracting the same quantity from each side of an inequality gives an equivalent inequality.

   -Example:

3. Multiplying each side of an inequality by the same positive quantity gives an equivalent inequality.

   -Example:

4. Multiplying each side of an inequality by the same negative quantity reverses the direction of the inequality.

   -Example:

5. Taking reciprocals of each side of an inequality involving positive quantities reverses the direction of the inequality.

   -Example:
6. Inequalities can be added.

**Example:**

**Linear Inequalities**  
An inequality is **linear** if each term is constant or a multiple of the variable.

To solve linear inequalities, we use the rules for inequalities given above to isolate the variable on one side of the inequality sign.

**Examples:**

(a) 

(b) 

**Nonlinear Inequalities**  
A **nonlinear** inequality involves squares and other powers of the variable.

**GUIDELINES FOR SOLVING NONLINEAR INEQUALITIES**

1. **Move all nonzero terms to one side.** If necessary, rewrite the inequality so that all nonzero terms appear on one side of the inequality sign. If the nonzero side of the inequality involves quotients, bring them to a common denominator.

2. **Factor.** Factor the nonzero side of the inequality completely.

3. **Find the intervals.** Set each factor equal to zero, and solve each resulting equation for the variable. Use these solutions to divide the real line into intervals.

4. **Make a table.** Use test values to make a table of the signs of each factor on each interval. In the last row of the table determine the sign of the product (or quotient) of these factors.
5. Solve. Determine the solution of the inequality from the last row of the sign table. Be sure to check whether the inequality is satisfied by some or all of the endpoints of the intervals.

-Examples: