Section 4.1 - Exponential Functions

• A function that has a variable in an exponent is called an exponential function.

Examples:

(a) \( f(x) = a^x \)

(b) \( f(x) = b^x \)

• For \( a > 0 \), the exponential function with base \( a \) is defined by \( f(x) = a^x \).

• Notice that the graph of any such function passes through the point \( (0, 1) \).

• If \( 0 < a < 1 \), the exponential function \( f(x) = a^x \).

• If \( a > 1 \), the exponential function \( f(x) = a^x \).

• The larger (or smaller) the base, the more rapid the increase (or decrease).

• For \( a \neq 1 \), the function \( f(x) = a^x \) has domain \( \mathbb{R} \) and range \( (0, \infty) \).

• If \( a = 1 \), the exponential function \( f(x) = 1^x \) with domain \( \mathbb{R} \) and range \( \{1\} \).
Identifying Graphs of Exponential Functions

Find the exponential function \( f(x) = a^x \) whose graph is given.

(a)  

(b)  

Transformations of Exponential Functions

(a)  

(b)  

(c)
Comparing Exponential and Power Functions: What Happens in the Long Run?

A **power function** is a function of the form \( f(x) = x^n \).

How do power functions and exponential functions compare as \( x \to \infty \)?

**Example:**

WINDOW 1:

WINDOW 2:

WINDOW 3:

- EVERY exponential growth function eventually dominates (overtakes) EVERY power function.

- EVERY exponential decay function will eventually approach 0 faster than EVERY power function with a negative exponent.

**The Natural Exponential Function**

- The **natural exponential function** is the exponential function \( f(x) = e^x \) with base \( e \). It is often referred to as the exponential function.

- Remember that \( e \approx 2.71828182845904523536 \).

**Examples:** Use the \( e^x \) key on your calculator to evaluate the following correct to five decimal places:

(a) 

(b) 

(c)
Examples of Graphing Exponential Functions with Base $e$

Example: The population of a certain species of bird is limited by the type of habitat required for nesting. The population behaves according to the growth model

$$n(t) = \frac{5600}{0.5 + 27.5e^{-0.04t}}$$

where $t$ is measured in years.

(a) Find the initial bird population.

(b) Find the bird population after 2 years.

(c) What size does the population approach as time goes on?