Section 3.2 - Dividing Polynomials

• The goal in this section is to factor certain polynomials completely.

• To accomplish this goal, we must first learn to divide polynomials. Two methods of dividing polynomials are
  1. Long Division
  2. Synthetic Division

• The polynomial that is being divided by another polynomial is called the **dividend**.

• The polynomial that we are dividing by is called the **divisor**.

**Long Division**

• Long division for polynomials is very similar to that for whole number division.

• When the divisor in long division is of degree greater than 1, the signal to stop the division process comes when the degree of the remainder is smaller than the degree of the divisor.

**Examples:**

(a)
Synthetic Division
Synthetic division can be used when the divisor is of the form $x - c$.

Examples:

(a)
The Factor Theorem

The real number $c$ is a zero of $P$ if and only if $x - c$ is a factor of $P(x)$.

Examples:

(a)

(b)
Section 3.6 - Rational Functions

- A **rational function** is a function of the form

\[ r(x) = \frac{P(x)}{Q(x)} \]

where \( P \) and \( Q \) are polynomials.

- Rational functions are not defined for those values of \( x \) for which the denominator is zero. When graphing a rational function, we must pay special attention to the behavior of the graph near those \( x \)-values.

**Example:**
Remember, for positive real numbers,

\[ \frac{1}{\text{BIG NUMBER}} = \text{small number} \]

and

\[ \frac{1}{\text{small number}} = \text{BIG NUMBER} \]

**Notation**
- \( x \to a^- \) means ________________________________.
- \( x \to a^+ \) means ________________________________.

**Definition of Asymptotes**

1. The line \( x = a \) is a **vertical asymptote** of the function \( y = f(x) \) if

\[ y \to \infty \text{ or } y \to -\infty \text{ as } x \to a^+ \text{ or } x \to a^- \]

2. The line \( y = b \) is a **horizontal asymptote** of the function \( y = f(x) \) if

\[ y \to b \text{ as } x \to \infty \text{ or } x \to -\infty \]

Informally speaking, an asymptote of a function is a line that the graph of the function gets closer and closer to as one travels along that line in either direction.

**Examples:**

(a) (b)