Section 5.1 - The Unit Circle

The **unit circle** is the circle of radius 1 centered at the origin in the $xy$-plane. Its equation is

$$x^2 + y^2 = 1$$

**Example:** Show that the given point is on the unit circle.

**Example:** The point $P$ is on the unit circle. Find $P(x, y)$ from the given information.

**Terminal Points on the Unit Circle**

- Suppose $t$ is a real number. The **terminal point** determined by the real number $t$ is the point $P(x, y)$ on the unit circle one arrives at by travelling a distance $t$ along the unit circle, starting at the point $(1, 0)$ and moving in a counterclockwise direction if $t$ is positive or in a clockwise direction if $t$ is negative.

- Remember that the circumference of the unit circle is ___________.

**Examples:** Find the terminal point on the unit circle determined by each real number $t$.

(a)  
(b)  
(c)  
(d)
Special Angles and Triangles

- We will soon show that
  - \( t = \frac{\pi}{6} \) corresponds to an angle with measure 30°
  - \( t = \frac{\pi}{4} \) corresponds to an angle with measure 45°
  - \( t = \frac{\pi}{3} \) corresponds to an angle with measure 60°

- Therefore the following triangles will be very useful:

30°-60°-90° Triangle          45°-45°-90° Triangle

- Use these triangles to find the terminal point determined by each given real number \( t \):

(a) \( t = \frac{\pi}{6} \)
The Reference Number
Let $t$ be a real number. The reference number $\bar{t}$ associated with $t$ is the shortest distance along the unit circle between the terminal point determined by $t$ and the $x$-axis.

Examples: Find the reference number for each value of $t$.

(a) $t = \frac{\pi}{4}$

(b) $t = \frac{\pi}{3}$

(c) $t = \frac{\pi}{6}$

(d) $t = \frac{3\pi}{4}$

Using Reference Numbers to Find Terminal Points
To find the terminal point $P$ determined by any value of $t$, we use the following steps:

1. Find the reference number $\bar{t}$.
2. Find the terminal point $Q(a,b)$ determined by $\bar{t}$.
3. The terminal point determined by $t$ is $P(\pm a, \pm b)$, where the signs are chosen according to the quadrant in which the terminal point lies.
Examples: Find the reference number $\overline{t}$ for each value of $t$, and use the reference number to find the terminal point determined by $t$. Label each point on your unit circle.

(a) $t = \frac{2\pi}{3}$  
(b) $t = \frac{4\pi}{3}$  
(c) $t = \frac{5\pi}{3}$

(d) $t = \frac{5\pi}{6}$  
(e) $t = \frac{7\pi}{6}$  
(f) $t = \frac{11\pi}{6}$

(g) $t = \frac{3\pi}{4}$  
(h) $t = \frac{5\pi}{4}$  
(i) $t = \frac{7\pi}{4}$

Examples: Find the terminal point determined by $t$.

(a)

(b)

(c)