Section 4.1 - Exponential Functions

- For $a > 0$, the exponential function with base $a$ is defined by $f(x) = a^x$.

- If $0 < a < 1$, the exponential function decreases from left to right (exponential decay) and has domain $(-\infty, \infty)$ and range $(0, \infty)$. The smaller the base, the more rapid the decrease.

- If $a = 1$, the exponential function is the horizontal line passing through the point $(0, 1)$.

- If $a > 1$, the exponential function increases from left to right (exponential growth) and has domain $(-\infty, \infty)$ and range $(0, \infty)$. The larger the base, the more rapid the increase.

Identifying Graphs of Exponential Functions
Find the exponential function $f(x) = a^x$ whose graph is given.

(a) 

Transformations of Exponential Functions

(a) 

(b)
Comparing Exponential and Power Functions: What Happens in the Long Run?

A power function is a function of the form \( f(x) = x^n \).

How do power functions and exponential functions compare as \( x \to \infty \)?

**Example:**

WINDOW 1:

WINDOW 2:

WINDOW 3:

- EVERY exponential growth function eventually dominates (overtakes) EVERY power function.

**Example:**

WINDOW 1:

WINDOW 2:

WINDOW 3:

- EVERY exponential decay function will eventually approach 0 faster than EVERY power function with a negative exponent.
The Natural Exponential Function
• The natural exponential function is the exponential function \( f(x) = e^x \) with base \( e \). It is often referred to as the exponential function.

• Remember that \( e \approx 2.71828182845904523536 \).

Examples: Use the \( e^x \) key on your calculator to evaluate the following correct to five decimal places:

(a)

(b)

(c)

Examples of Graphing Exponential Functions with Base \( e \)
Example: The population of a certain species of bird is limited by the type of habitat required for nesting. The population behaves according to the growth model

\[ n(t) = \frac{5600}{0.5 + 27.5e^{-0.044t}} \]

where \( t \) is measured in years.

(a) Find the initial bird population.

(b) Find the bird population after 2 years.

(c) What size does the population approach as time goes on?